KiboCUBE Academy

Lecture 11

Introduction to Nano-Satellite Structures

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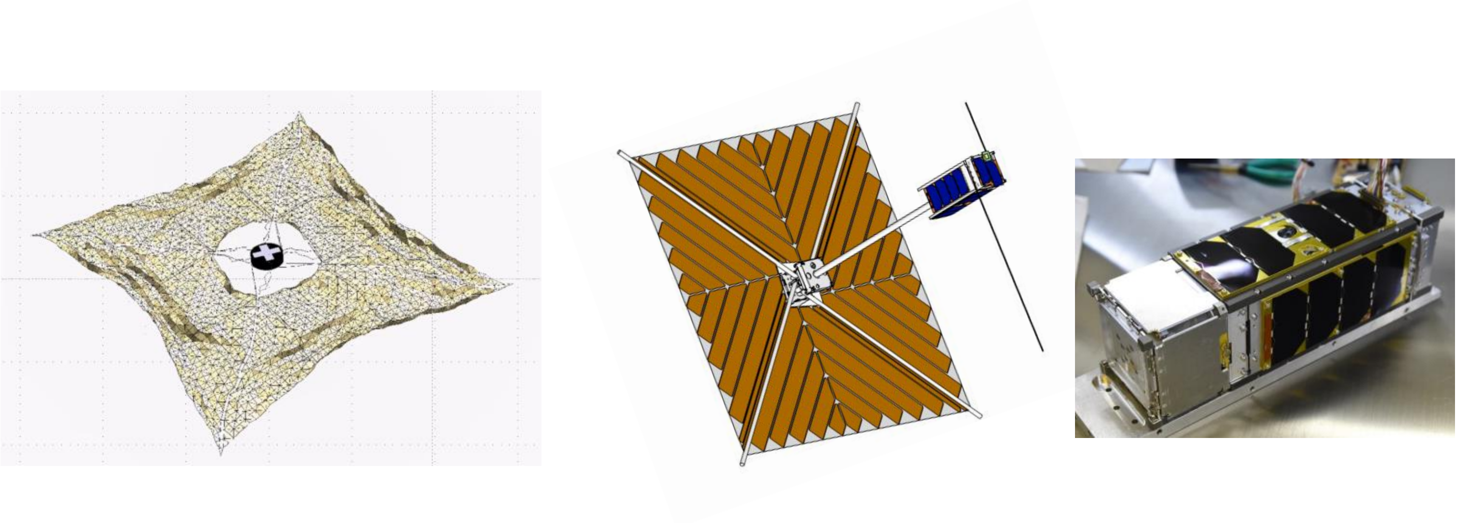
Department of Mechanical Engineering
Associate Professor Hiraku SAKAMOTO, Ph.D.

This lecture is NOT specifically about KiboCUBE and covers GENERAL engineering topics of space development and utilization for CubeSats.
The specific information and requirements for applying to KiboCUBE can be found at:
https://www.unoosa.org/oosa/en/ourwork/psa/hsti/kibocube.html

Lecturer Introduction



Hiraku SAKAMOTO, Ph.D.

Position:

2015- Associate Professor
Department of Mechanical Engineering, Tokyo Institute of Technology, Japan.
2015- Board Member, University Space Engineering Consortium Japan (UNISEC)
2015-2019 Principal Investigator, 3U CubeSat **OrigamiSat-1/FO-98**

Research Topics:

Space deployable structures, Systems engineering for small spacecraft development and utilization

Contents

1. Introduction

1. Introduction

1.1 Introduction to nano-satellite structures

Structure Engineers’ mind

Systems Engineering

Magnetization

Grounding for avionics

Thermal balance

Thermal deformation

Alignment stability

Rocket interface
- Mechanical
- Electrical

System integration and test

Debris protection

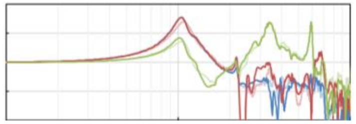
Radiation protection

Buckling

$$F = \frac{\pi^2 EI}{(KL)^2}$$

Dynamic stiffness

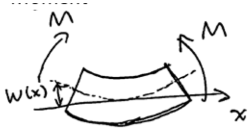
- Resonance



Component assembly
- **Accessibility**
- Tools

Bending deformation

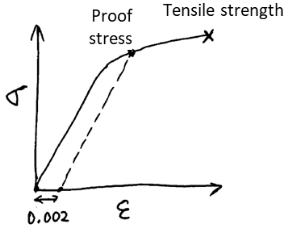
$$M = -EI\frac{d^2w}{dx^2}$$

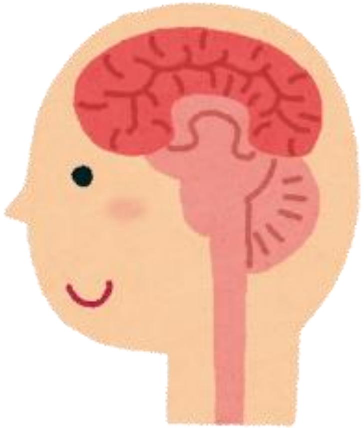


Mass vs Strength/Stiffness

Yield strength

$$\sigma = E\varepsilon$$





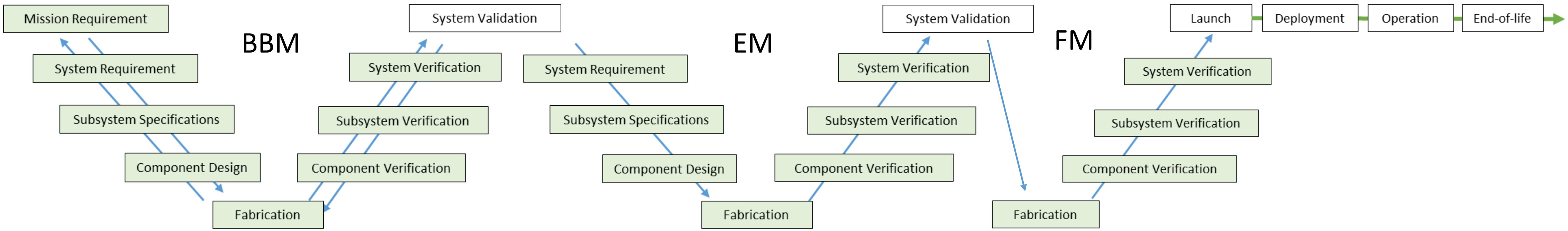
Design verification
- Analysis (FEM)
- Test

Material
- Ease of **manufacture**
- Erosion by atomic oxygen

1. Introduction

1.2 Structures in spacecraft’s lifecycle (1/2)

Typical mission lifecycle



Mission Requirement

BBM

System Requirement

Subsystem Specifications

Component Design

Fabrication

Component Verification

Subsystem Verification

System Verification

System Validation

EM

System Requirement

Subsystem Specifications

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FM

Fabrication

Component Verification

Subsystem Verification

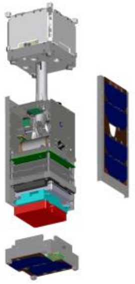
System Verification

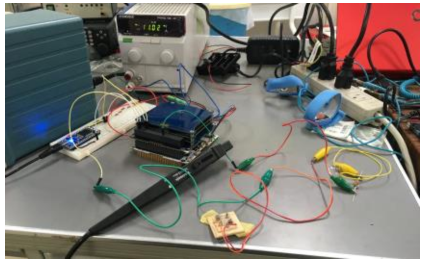
Launch

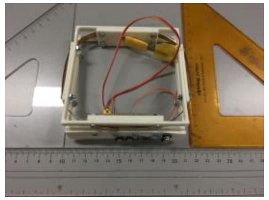
Deployment

Operation

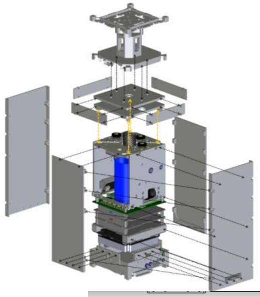
End-of-life

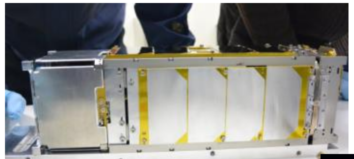


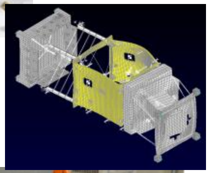




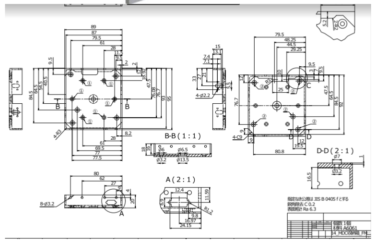
- ✓ CAD drawing
- ✓ Mock-up

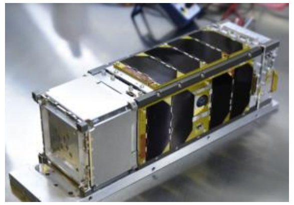




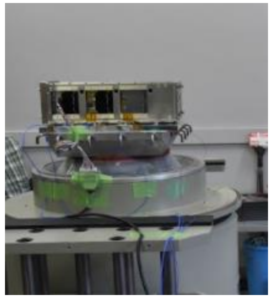


- ✓ Machining drawing
- ✓ Structural analysis
- ✓ EM structure





- ✓ FM

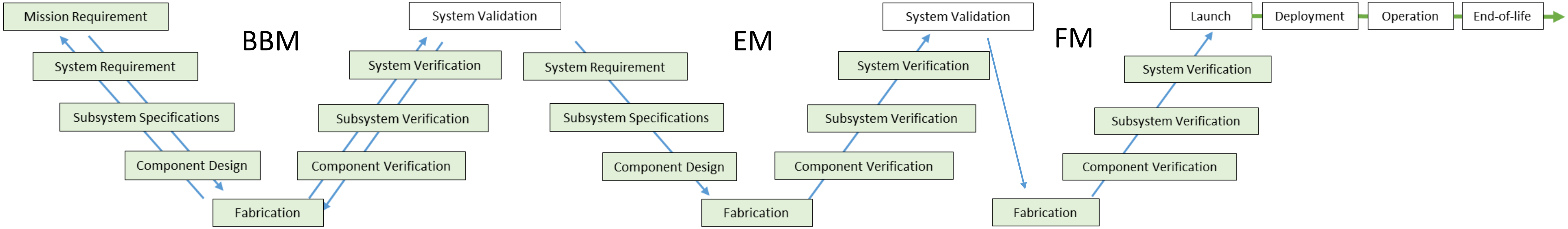


1. Introduction

1.2 Structures in spacecraft's lifecycle (2/2)

A **spacecraft's structure** is its underlying body, tasked with keeping the spacecraft **suitably rigid** to support its instruments and subsystems. (ESA, *Enabling & Support* [1])

[1] https://www.esa.int/Enabling_Support/



Mission Requirement
System Requirement
Subsystem Specifications
Component Design
Fabrication
Component Verification
Subsystem Verification
System Verification
System Validation

BBM

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EM

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Component Verification
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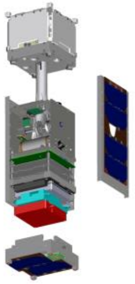
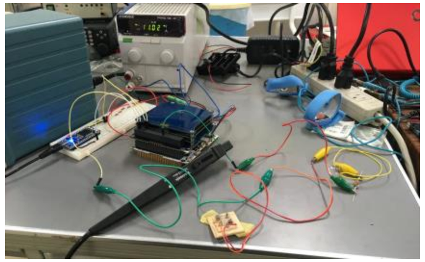
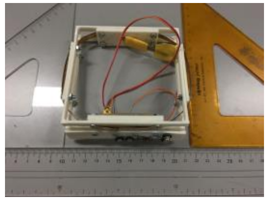
System Verification

FM

Launch
Deployment
Operation
End-of-life







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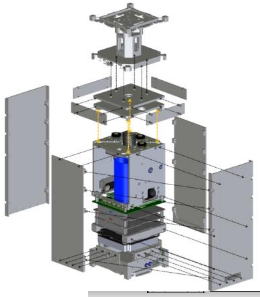

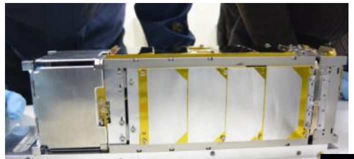

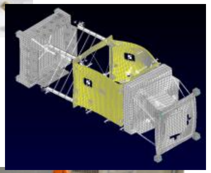

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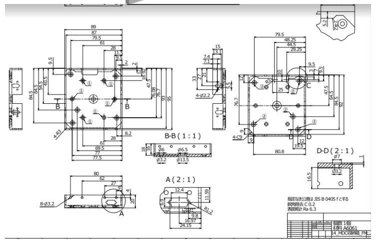

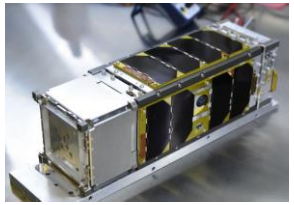

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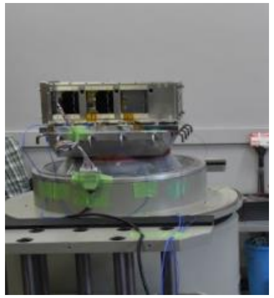
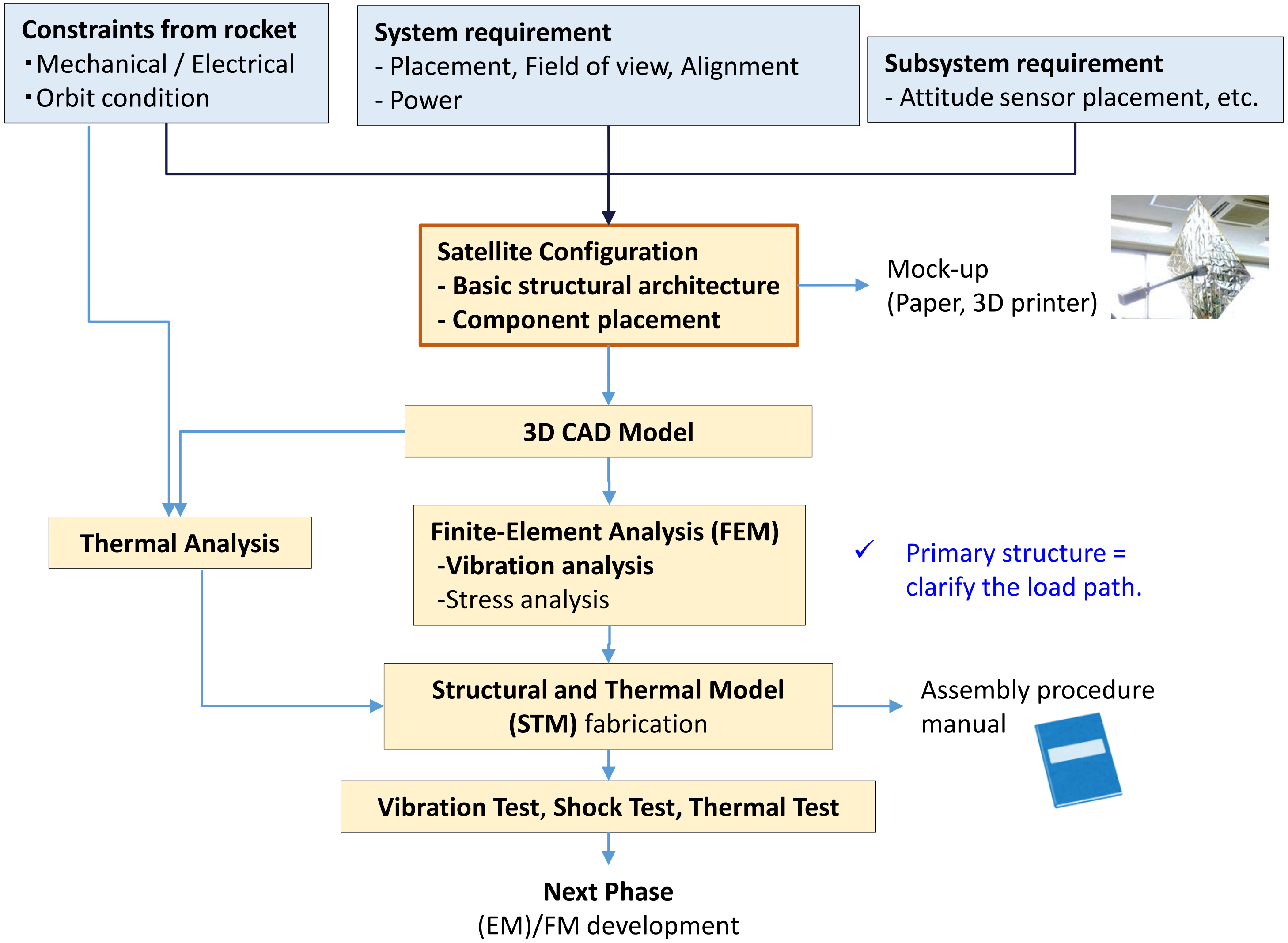


1. Introduction

1.3 Design process for nano-satellite structures



Constraints from rocket
- Mechanical / Electrical
- Orbit condition

System requirement
- Placement, Field of view, Alignment
- Power

Subsystem requirement
- Attitude sensor placement, etc.

Satellite Configuration
- Basic structural architecture
- Component placement

Mock-up
(Paper, 3D printer)

3D CAD Model

Thermal Analysis

Finite-Element Analysis (FEM)
-**Vibration analysis**
-Stress analysis

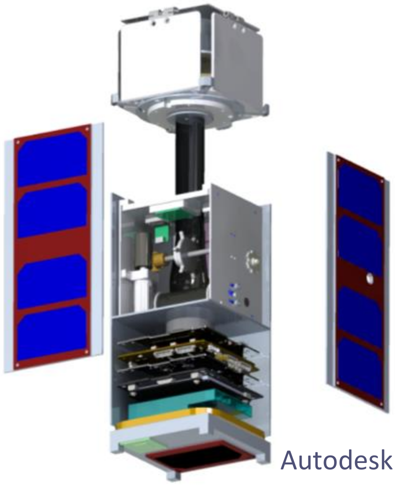
✓ Primary structure = clarify the load path.

Structural and Thermal Model (STM) fabrication

Assembly procedure manual

Vibration Test, **Shock Test, Thermal Test**

Next Phase
(EM)/FM development



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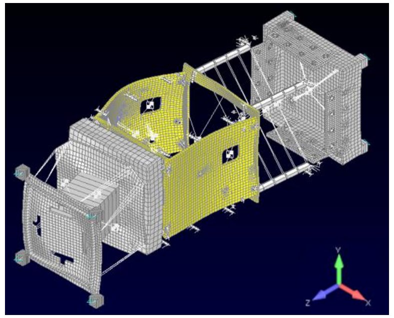
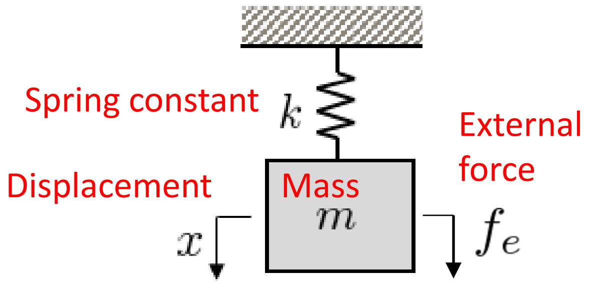


Femap with Nastran®

1.4 Concept of natural frequencies (1/2)



<u>Let's derive the natural</u> <u>(angular)</u> **<u>frequency:</u>** $\Omega = \sqrt{\frac{k}{m}}$ [rad/s]

Equation of motion (Newton's 2nd law) $m\ddot{x} + kx = f_e$

Inertial force Spring force (Hook's law)

Assume **harmonic force & response**

ω: excitation angular frequency

External force $f_e = Fe^{j\omega t}$ Displacement $x = Xe^{j\omega t}$

$F,\ X$: constant t: time

j: imaginary number

Note: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$ Euler's formula

Substitute into equation of motion $-\omega^2 mXe^{j\omega t} + kXe^{j\omega t} = Fe^{j\omega t}$

$$\boxed{\frac{X}{F} = \frac{1}{k - \omega^2 m}}$$

Compliance **Frequency Response Function (FRF)**

When $\omega \rightarrow \sqrt{\frac{k}{m}}$, $\underline{\left|\frac{X}{F}\right| \rightarrow \infty}$

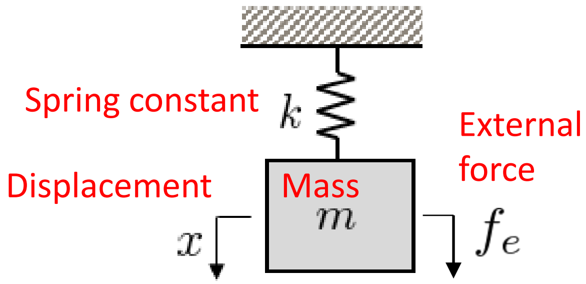
Resonance

Thus, $\Omega = \sqrt{\frac{k}{m}}$ [rad/s]

✓ With **natural frequency** excitation, displacement is large!!

1. Introduction

1.4 Concept of natural frequencies (1/2)



Unit:

Angular frequency

$$\omega, \Omega \ [\mathrm{rad/s}]$$

Frequency

$$f = \frac{\omega}{2\pi} \ [\mathrm{Hz}]$$

How many periods in 1 second?

Let's derive the natural (angular) **frequency:** $\Omega = \sqrt{\frac{k}{m}} \ [\mathrm{rad/s}]$

Equation of motion (Newton's 2nd law) $m\ddot{x} + kx = f_e$

Inertial force Spring force (Hook's law)

Assume **harmonic force & response** External force $f_e = Fe^{j\omega t}$ Displacement $x = Xe^{j\omega t}$

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Note: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$ Euler's formula

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When $\omega \rightarrow \sqrt{\frac{k}{m}}$, $\left|\frac{X}{F}\right| \rightarrow \infty$

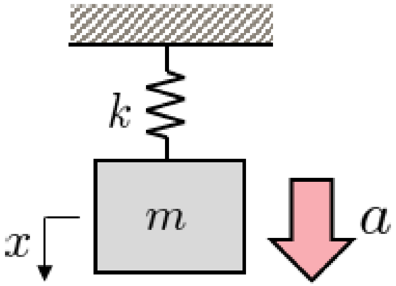
Resonance

Thus, $\Omega = \sqrt{\frac{k}{m}} \ [\mathrm{rad/s}]$

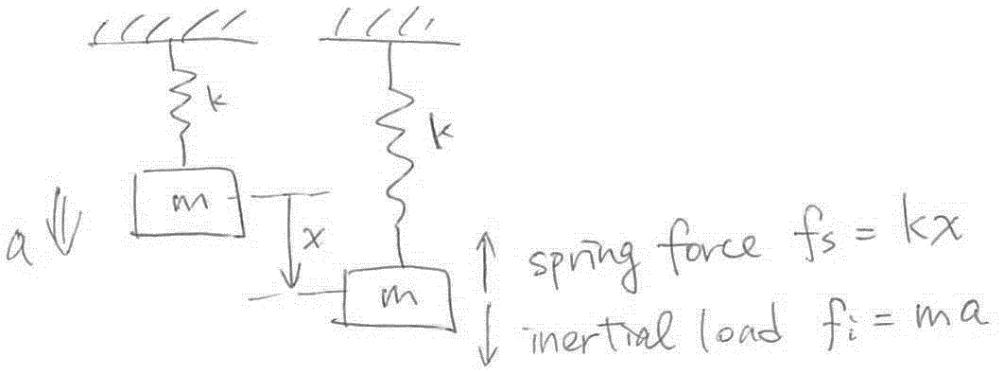
✓ With **natural frequency** excitation, displacement is large!!

1. Introduction

1.4 Concept of natural frequencies (2/2)



Why natural frequencies of structures are so important for spacecraft design?



Force balance: $f_s = f_i$ d'Alembert's principle

$$kx = ma$$

Natural (angular) frequency

$$\boxed{\Omega = \sqrt{\frac{k}{m}}\ [\mathrm{rad/s}]}$$

Thus, Displacement $x = \frac{m}{k}a = \underline{\frac{a}{\Omega^2}}$

When disturbance is acceleration,
system's stiffness is represented by **"natural frequency squared".**

∴ Generally spacecraft's stiffness is evaluated by natural frequencies.
(e.g. Axial direction requirement:
large satellite 30Hz, small satellite 100Hz)

2. Important Topics in Nano-satellite Structures

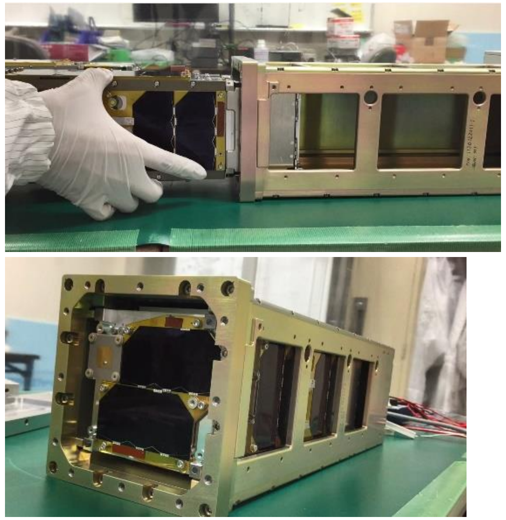
2. Important Topics in Nano-satellite Structures

2.1 Mass properties and dimensions

Mass:

- Mass should be properly managed in tables, and gradually shifted from a **"rough estimate"** to a **"highly accurate actual measurement"** during the development process.
  - The latest mass should be carefully traced.
- Initially, **a large mass margin** of about 10% should be kept.
  - If the mass exceeds the margin, there is a risk of significant design change such as reducing the stiffness and strength of **primary structure**.

Inertial property:



- Center-of-mass location, Moment of inertia, and Products of inertia
  - Managed by 3D CAD. Actual measurements may be required.

Dimension (size):

- Structure should be simple to enable high repeatability of assembly accuracy.
  - Spacecraft are often disassembled and reassembled frequently during development process.

2. Important Topics in Nano-satellite Structures

2.2 Rocket interfaces

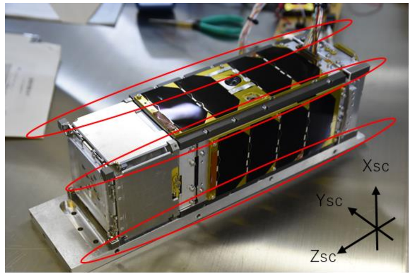
Check the interface control documents (ICDs) with your launch vehicle carefully.

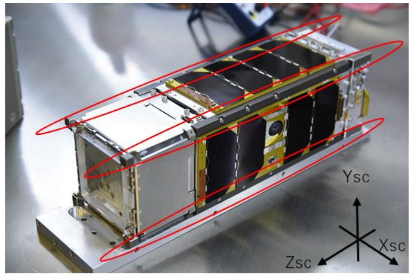
Mechanical Interface:

- Surface roughness, strength, coating
  - CubeSat's rails are normally **hard-anodized.** (Epsilon rocket requires MIL-A-8625 "Anodic Coatings for Aluminum and Aluminum Alloys" Type3, thickness over 10μm)
- Deployment switch
- Separation mechanisms
- Venting
  - During launch, the internal volume of air should safely evacuate through enough venting holes. (Avoid a closed container!)

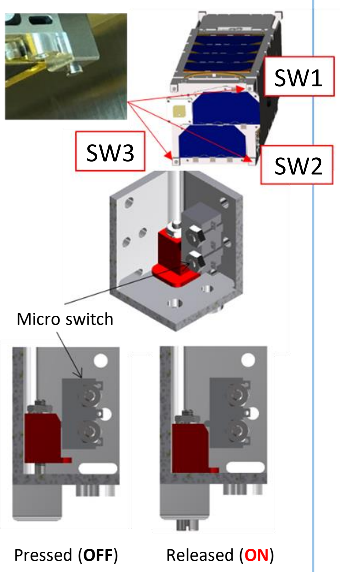
Electrical Interface:

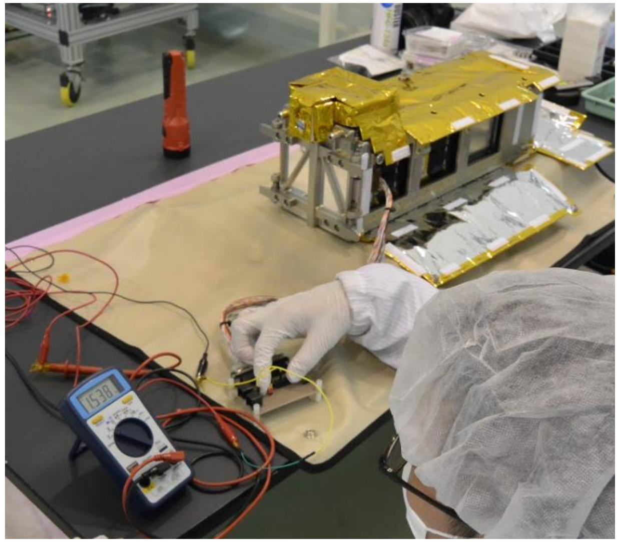
- Umbilical connector





Deployment switches





2. Important Topics in Nano-satellite Structures

2.3 Applied loads (1/2)

Launch loads: often categorized into four types with different frequency domain.

(1) Quasi-static acceleration load:

- The acceleration in the direction of flight caused by the engine's thrust (about 10G).

(2) Sine vibration load:

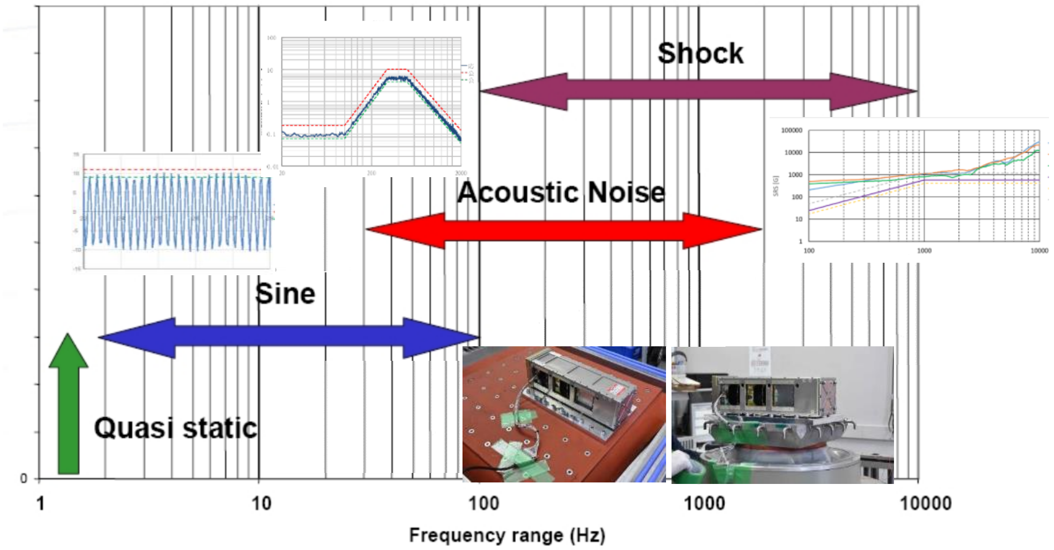
- Caused by rocket vibration, with a large peak at 10 to 30 Hz, under 100Hz.

(3) Random vibration load = acoustic load:

- The acoustic vibrations from the engine jets are propagated from the mechanical interface.
- Additionally, in the atmosphere, acoustic loads are applied directly through the fairing.

(4) Shock load:

- Shock caused by explosions of pyrotechnics and separation of the spacecraft.



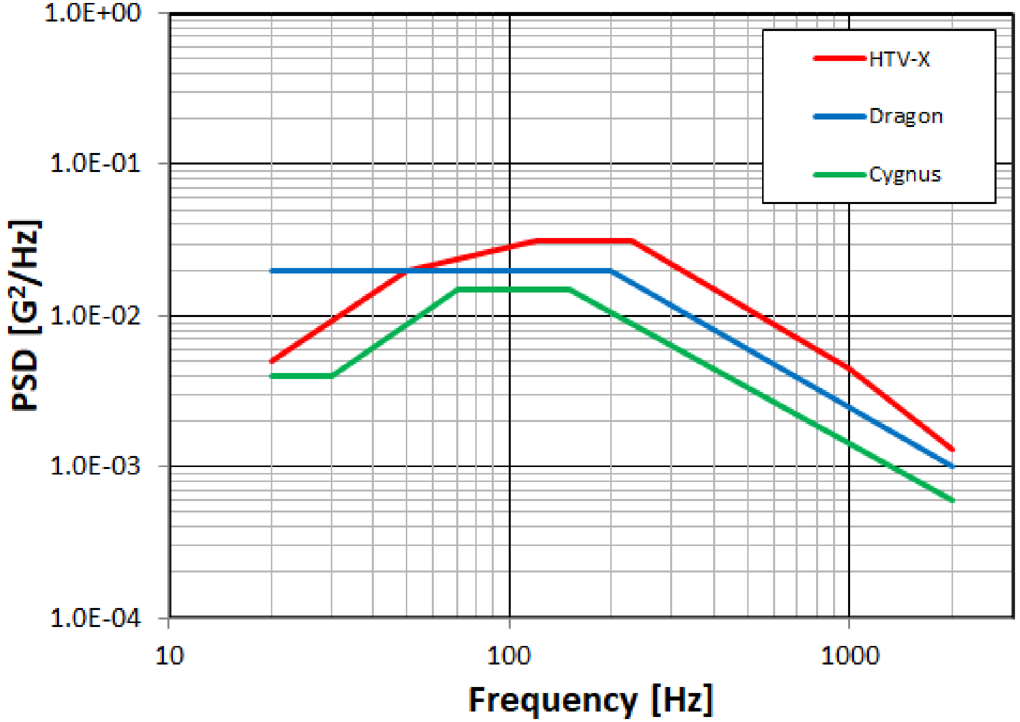
Appendix: Slide from Prof. Kuwahara's lecture

Launch Conditions of KiboCUBE's Launch Vehicles

- Random vibration condition

HTV-X		Dragon		Cygnus	
Freq. (Hz)	PSD (g^2/Hz)	Freq. (Hz)	PSD (g^2/Hz)	Freq. (Hz)	PSD (g^2/Hz)
20	0.005	20	0.02	20	0.004
50	0.02	200	0.02	30	0.004
120	0.031	2000	0.001	70	0.015
230	0.031			150	0.015
1000	0.0045			2000	0.0006
2000	0.0013				
Overall (grms)	4.05	Overall (grms)	3.2	Overall (grms)	2.44
Duration (sec)	60	Duration (sec)	60	Duration (sec)	60

Random vibration conditions of launch vehicles



- Quasi-static acceleration condition
  - HTV-X: 6.0 [g]
  - SpaceX Dragon: 9.0 [g]
  - Orbital Cygnus: 9.0 [g]

- Shock condition
  - N/A

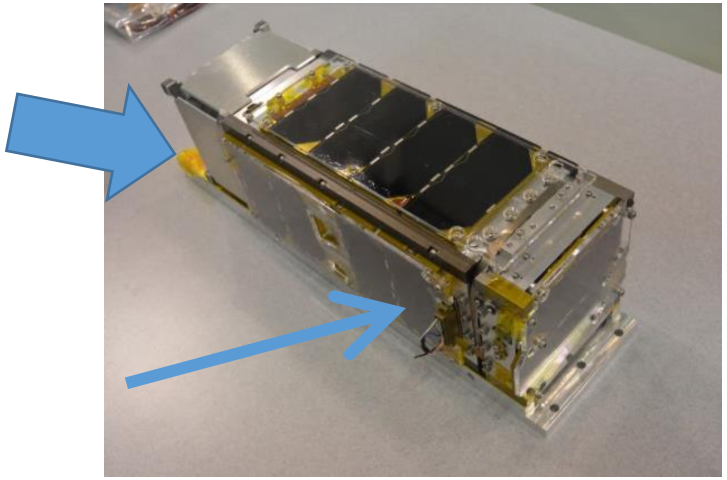
Reference: JEM Payload Accommodation Handbook Vol. 8 D (Japanese)
https://iss.jaxa.jp/kibouser/library/item/jx-espc_8d.pdf

2. Important Topics in Nano-satellite Structures

2.3 Applied loads (2/2)

Load applied during handling on ground:

Watch out for stress concentration!



Handles for ground transportation

- Inevitably causes concentrated loading. Check the **safety factor**.
- Would a CubeSat be hand-held?　How?

Support jig for various testing

- Assume rotating to various orientations

Protective cover for solar cells

- Need holes on satellites for attachment

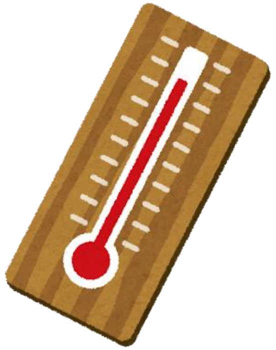
Container for transportation

- Can safe transportation be achieved?

2. Important Topics in Nano-satellite Structures

2.4 Thermal requirements

Keeping within an allowable temperature range



- **Conduction:** Thermal contact resistance → Use heat-conducting gap filler at fasteners to reduce uncertainty.
- **Radiation:** Adjust surface absorptivity **α** and emissivity **ε**
  - ➢ Surface treatment, MLI attachment, etc.
- **Addition of heaters**, etc.

Thermal deformation

- **Strength:** Thermal stress should be within the structural strength.
  - ➢ Watch for fatigue due to thermal cycling
- **Alignment** should be satisfied even with thermal deformation

Thermal test using Structural and Thermal Model (STM) (or Engineering Model (EM))

- **Thermal balance test:** Main purpose is to correlate the thermal mathematical model.
- **Thermal vacuum test:** Verify satellite functions with highest and lowest satellite temperatures.

2. Important Topics in Nano-satellite Structures

2.5 Materials

Physical and mechanical properties of materials (Young's modulus, allowable stress, etc.) are defined by reliable documents such as

- MMPSD (Metallic Materials Properties Development and Standardization)
- MIL-HDBK-5J, MIL-HDBK-17

Use materials with good workability, high specific stiffness (E/ρ), and high specific strength (σ_{max}/ρ)

- Aluminum alloys are most commonly used.
  - The 6061-T6 alloy is slightly lower in strength, but easier to fabricate.
  - 7075-T6 material is desirable in terms of strength, stiffness, workability, and availability, but has stress corrosion problems.
- Carbon composite materials have high specific stiffness and strength.
- Titanium, tungsten, etc. are not recommended because they are difficult to melt during re-entry into the Earth's atmosphere (JERG-0-002-HB002).
- Polymer materials are susceptible to Atomic Oxygen (AO) in low earth orbit (easy to degrade)

Be careful of contamination by outgassing.

- Due to the high vacuum and high temperature, gas is generated by some materials. This gas agglomerates on the surface of instrument. = Contamination
  - Causing degradation of optical devices, lowering of the power generated by solar cells
- **JAXA Material Database**: Search for outgas data https://matdb.jaxa.jp/Outgas/OG_search_e.html

➡ Bake-out before using high-spec Thermal Vacuum chamber

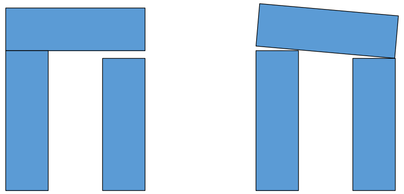
2. Important Topics in Nano-satellite Structures

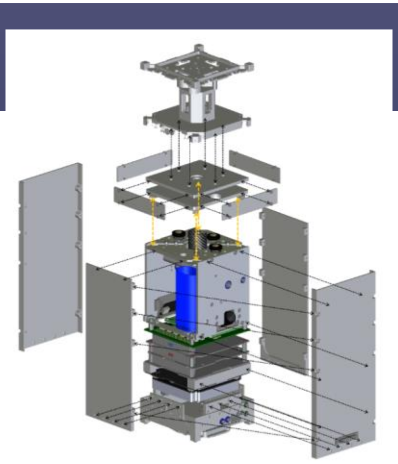
2.6 **Fabrication** and assembly (1/2)

Use as simple structure as possible.

  - Very important!!!

- Specify the following in the drawing:
  - ➢ **Dimensional tolerance**
  - ➢ **Surface roughness**





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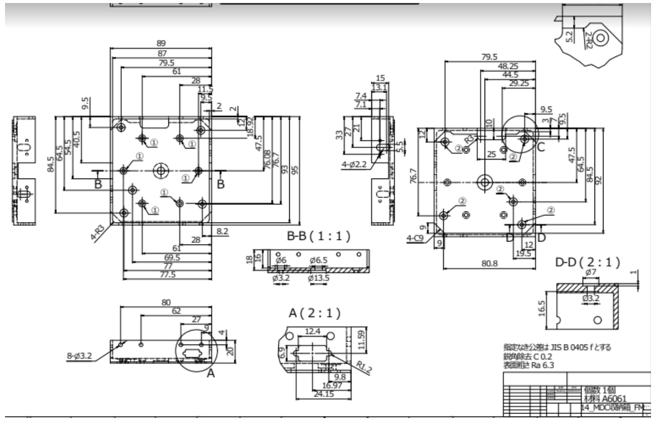


- Clean machining oil, etc. **to avoid contamination.**
- Consider **storage of your parts** to prevent scratches after machining.
- Common **surface treatments**
  - **Hard anodizing treatment:** Forms a hard, wear-resistant oxide layer on aluminum surfaces. Non-conductive (insulating) property.
  - **Alodine treatment:** Increases corrosion resistance of aluminum alloys. Conductive.
  - **Molybdenum disulfide (MoS_2) coating:** Used for sliding parts as solid lubrication.

2. Important Topics in Nano-satellite Structures

2.6 Fabrication and **assembly** (2/2)

Accessibility:

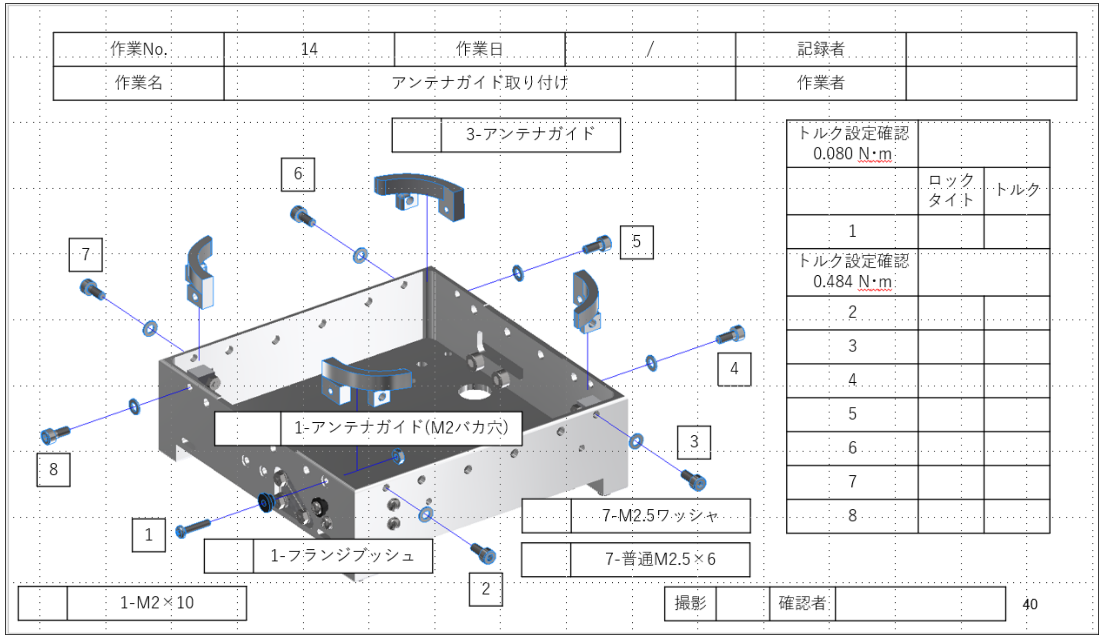
- Does the design accommodate assembly tools?
- Design and manufacture jigs to support assembly.
- Create and update **assembly procedure document** to improve reproducibility.

Simple assembly is desirable.

- As many subsystems as possible should be accessible after assembly.
- Subsystem replacement should be possible with minimal disassembly.

Fasteners (bolts, nuts, washers, etc.)

- Based on standards such as JIS, MIL.
- Tightening torque shall be controlled by using a torque wrench/torque driver.
  - ➢ Improper tightening torque increases risk of loosening or breakage.
- Use anti-loosening measures.
  - ➢ Spring washers
  - ➢ Thread lock adhesive (low outgas)
  - ➢ Double nut
  - ➢ Fixing with wires, pins, etc.



作業No.	14	作業日	/	記録者	
作業名	アンテナガイド取り付け			作業者	

トルク設定確認 0.080 N･m		
	ロック タイト	トルク
1		
トルク設定確認 0.484 N･m		
2		
3		
4		
5		
6		
7		
8		

3-アンテナガイド

1-アンテナガイド(M2バカ穴)

1-フランジブッシュ

7-M2.5ワッシャ

7-普通M2.5×6

1-M2×10

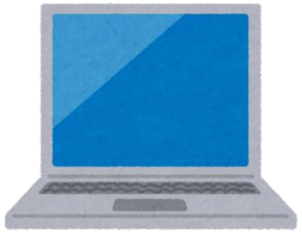
撮影		確認者	

40

2. Important Topics in Nano-satellite Structures

2.7 Analysis methods

1) **Back-of-envelope calculation**
    - ➢ Refer to theories in Section 3.1-3.3.
2) **Finite Element Method (FEM)**
    - ➢ Various commercial software available
    - ➢ Refer to Section 3.4.



Good structural design facilitates **analysis:**

- Use simple-shaped members.
- Each structural element should be designed to carry a single load path as much as possible (e.g. axial load member, shear-only member, etc.).
- Load path should be unique
    - ➢ Uncertainty significantly reduces accuracy of analysis
- Separate **primary structure** from secondary structure.
- Structural analysis with gaps is difficult. ∴ Analysis is easy if structure is rigidly fastened.

2. Important Topics in Nano-satellite Structures

2.8 Test methods

(More details are in Ch. 4.)

Design development test

The purpose is to obtain technical data for designing.

Often conducted for components and subsystems alone.

1. Evaluation of design feasibility
2. Validation of analysis methods
3. Establishment of test methods
4. Clarification of failure modes, etc.

Qualification/Acceptance test

In the case of EM/FM development, the following are conducted:

- **Qualification Test (QT):** Test under more severe conditions than flight to demonstrate that the satellite meets the requirement specification with an appropriate margin.
  (e.g. √2 times the load for vibration test, or 2 times the number of shocks for shock test, etc.)
- **Acceptance Test (AT):** The design has already been validated by QT, but fabrication of the flight model has not been validated; thus, the test is conducted under the conditions as expected in flight to screen workmanship error.

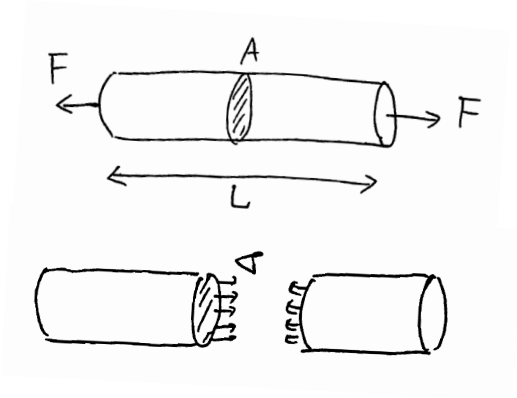
3. Theories for Structure Design

3. Theories for Structure Design

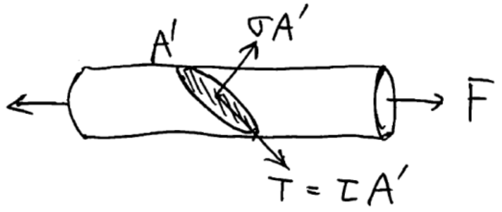
3.1 Mechanics of materials (1/2)

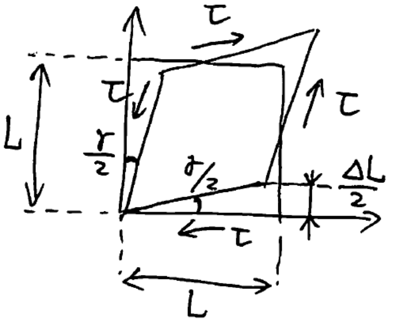
Elastic modulus, bending deformation

- Stress $\sigma = \frac{F}{A}$
- Strain $\varepsilon = \frac{\Delta L}{L}$
- Hook’s law $\sigma = E\varepsilon$
- Young’s modulus E

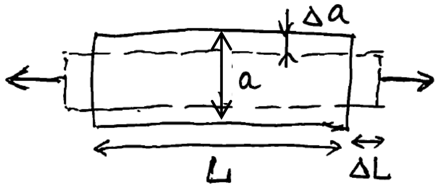


- Shear stress $\tau = \frac{T}{A'}$
- Shear strain $\gamma = \frac{\Delta L}{L}$
- Hook’s law in shear direction $\tau = G\gamma$
- Shear modulus of elasticity G





- Poisson's ratio ν



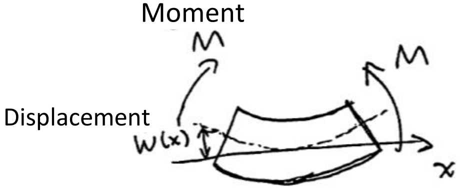
$$\frac{\Delta a}{a} = -\nu \frac{\Delta L}{L}$$

3 parameters (E, ν, G) are called **Elastic Moduli**

- For isotropic material, $G = \frac{E}{2(1+\nu)}$

Bending deformation of Beam

Long and slender, but bending stiffness is not negligible



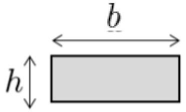
$$M = -EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

EI = “bending stiffness”

Important

For rectangular cross-section

- 2nd moment of area $I = \frac{bh^3}{12}$



I depends on cross-sectional shape

3. Theories for Structure Design

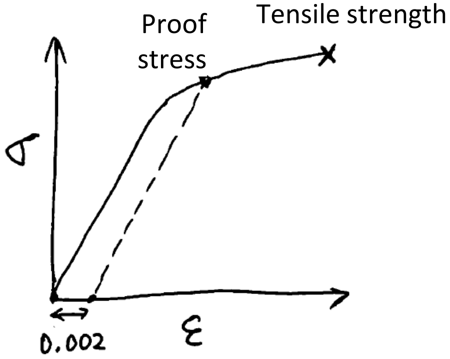
3.1 Mechanics of materials (2/2)

Failure mode (strength), safety factor

Yielding, Fracture

For metallic materials

- ✓ Permanent strain of 0.2% remains = "**yield**". Stress at this time = "proof stress".
- ✓ When the stress exceeds the tensile strength, the material **fractures**.



von Mises stress (= yielding index)

- ✓ In a 3D coordinate, there 6 stress parameters (complicated!).
- ✓ Therefore, one value is used as an index to evaluate risk of yielding.

$$\sigma_M = \sqrt{3J_2}$$

von Mises stress
(Always non-negative either in tension/compression)

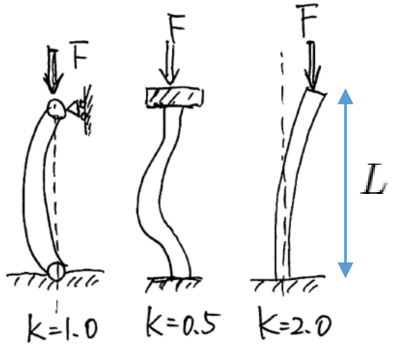
$$J_2 = \frac{1}{6}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2\right]+\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2$$

Buckling

For **column** structures,

- ✓ In axial compression, a certain load (**buckling load**) causes deformation to jump to "bending".

$$F = \frac{\pi^2 EI}{(KL)^2}$$



Failure mode: yielding, fracture, buckling, etc.

Factor of Safety (FS), Margin of Safety (MS)

Proof stress = **(FS)** x design stress

Tensile strength (ultimate stress) = **(FS)** x design stress

Often use a safety factor of 1.25 to 1.5 to calculate MS.

Margin of Safety: **MS**

$$\text{MS for yielding} = \frac{\text{Yielding stress}}{\textbf{FS}\text{ x Expected stress}} - 1$$

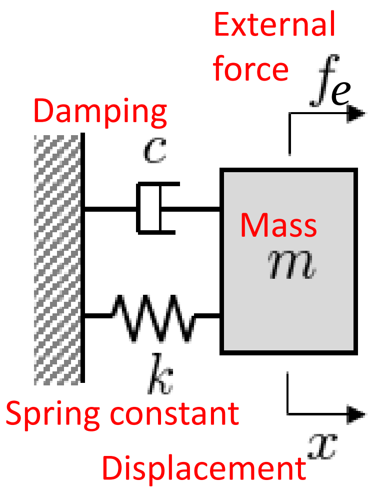
✓ **positive**

$$\text{MS for fracture} = \frac{\text{Ultimate stress}}{\textbf{FS}\text{ x Expected stress}} - 1$$

3. Theories for Structure Design

3.2 Mechanics of vibrations (1/4)

The vibration amplitude at resonance varies significantly depending on the “**damping ratio**”.



1 degree-of-freedom (DOF) vibration system with viscous damping

$$m\ddot{x} + \boxed{c\dot{x}} + kx = f_e$$

Damping force

Assume **harmonic force & response**

ω: excitation angular frequency

External force $f_e = Fe^{j\omega t}$

Displacement $x = Xe^{j\omega t}$

$F,\ X$: constant

Substitution gives

$$\left(-m\omega^2 + jc\omega + k\right) X = F \xrightarrow{\quad} \left(-\omega^2 + 2j\zeta\Omega\omega + \Omega^2\right) X = \frac{F}{m}$$

Damping ratio $\boxed{\zeta = \dfrac{c}{2\sqrt{mk}}}$

Natural (angular) frequency $\Omega = \sqrt{\dfrac{k}{m}}$

Therefore,

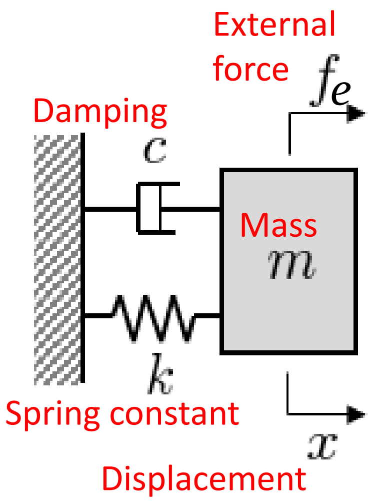
$$\boxed{\frac{X}{F} = \frac{1/m}{-\omega^2 + 2j\zeta\Omega\omega + \Omega^2}}$$

Compliance **Frequency Response Function (FRF)** (with damping effect)

3. Theories for Structure Design

3.2 Mechanics of vibrations (2/4)

Understand “Bode plot” that visualizes vibrational amplitudes and phases



Compliance **Frequency Response Function (FRF)** in the previous page is complex number (=2D information).

Gain (amplitude ratio):
$$\left|\frac{X}{F}\right| = \frac{1/m}{\sqrt{(\Omega^2-\omega^2)^2+(2\zeta\Omega\,\omega)^2}}$$

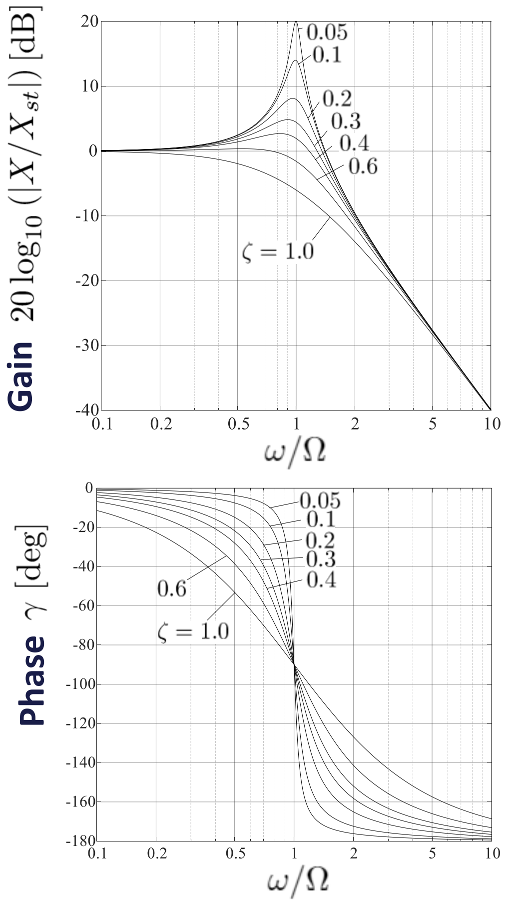
Phase:
$$\gamma = \tan^{-1}\left(\frac{-2\zeta\Omega\omega}{\Omega^2-\omega^2}\right)$$

Difference of timing between input and output

In the Bode plot on the right, the horizontal axis is ω/Ω .
i.e. The excitation frequency is normalized by the natural frequency.
(The horizontal axis shows the ratio of the excitation frequency to the natural frequency.)

In addition, the vertical axis of the gain is normalized by $X_{st}=\dfrac{F}{k}$ and displayed in decibels [dB].

Bode plot (visualization of frequency response)

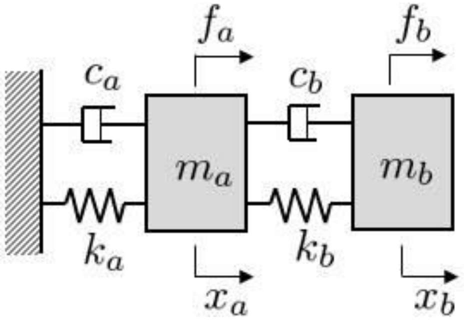


3. Theories for Structure Design

3.2 Mechanics of vibrations (3/4)

Multi-DOF system is represented by superposition of a single-DOF system.

How many natural frequencies for 2DOF system? **= 2.**



If there is no damping $(c_a = c_b = 0)$
the equation of motion (EOM) is written as

$$\begin{bmatrix} m_a & 0 \\ 0 & m_b \end{bmatrix} \begin{bmatrix} \ddot{x}_a \\ \ddot{x}_b \end{bmatrix} + \begin{bmatrix} k_a + k_b & -k_b \\ -k_b & k_b \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} = \begin{bmatrix} f_a \\ f_b \end{bmatrix}$$

Let's obtain natural frequencies of this 2DOF system. When the external forces are zero, matrix representation of EOM is

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = 0$$

For harmonic solution, assume

$$\boldsymbol{x}(t) = \begin{bmatrix} x_a(t) \\ x_b(t) \end{bmatrix} = \begin{bmatrix} X_a \\ X_b \end{bmatrix} e^{j\omega t} = \boldsymbol{X} e^{j\omega t}$$

Substitution yields

$$\left(-\omega^2 \boldsymbol{M} + \boldsymbol{K}\right) \boldsymbol{X} = 0$$

The condition to have a non-zero displacement solution is, from the knowledge of linear algebra,

$$\det\left(-\omega^2 \boldsymbol{M} + \boldsymbol{K}\right) = 0$$

This is actually an **eigen-value problem of a 2x2 matrix**.

$$\left(\boldsymbol{M}^{-1}\boldsymbol{K}\right) \boldsymbol{X} = \omega^2 \boldsymbol{X}$$

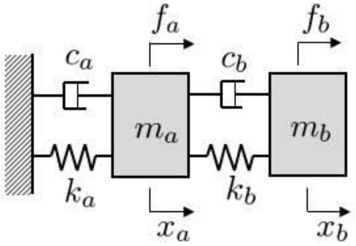
Thus, there are usually two eigen values. Two eigen values corresponds to two natural frequencies: Ω_1^2, Ω_2^2

3. Theories for Structure Design

3.2 Mechanics of vibrations (4/4)

Multi-DOF system is represented by the superposition of single-DOF system.

Each **vibration mode** in a multi-DOF system can be considered as a 1DOF system.



Two eigen vectors: $\phi_1, \ \phi_2$
are called vibration "**mode shapes**" herein.

Matrix form: $\Phi = [\phi_1 \ \phi_2]$

Diagonalization of EOM

$$\Phi^T M \Phi = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\Phi^T K \Phi = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

Scaling of mode shape vector: $\Psi = [\psi_1 \ \psi_2] = \begin{bmatrix} \frac{\phi_1}{\sqrt{m_1}} & \frac{\phi_2}{\sqrt{m_2}} \end{bmatrix}$

Then,

$$\Psi^T M \Psi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \Psi^T K \Psi = \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix}$$

Coordinate transformation: $\begin{bmatrix} x_a(t) \\ x_b(t) \end{bmatrix} = [\psi_1 \ \psi_2] \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \Psi \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix}$

Modal coordinate

EOM is completely decoupled.

$$\boxed{\begin{bmatrix} \ddot{\xi}_1 + \Omega_1^2 \xi_1 \\ \ddot{\xi}_2 + \Omega_2^2 \xi_2 \end{bmatrix} = \Psi^T \begin{bmatrix} f_a \\ f_b \end{bmatrix}}$$

The same is true for general ***n* DOF system**.
Compliance Frequency Response Function (FRF) for ***n* DOF system is:**

$$\boxed{G_{qr}(\omega) = \frac{X_q}{F_r} = \sum_{\gamma=1}^{n} \frac{\psi_{\gamma q}\psi_{\gamma r}}{-\omega^2 + 2j\zeta_\gamma \Omega_\gamma \omega + \Omega_\gamma^2}}$$

where $q, r = 1, 2, \ldots, n$ $\qquad \psi_\gamma = [\psi_{\gamma 1} \ \psi_{\gamma 2} \ \cdots \ \psi_{\gamma n}]^T$

DOF number

Mode shape vectors

3. Theories for Structure Design

3.3 Dynamics of solid structures

Equation of motion for a beam (continuum) is a partial differential equation.

Similarly as before, **natural frequencies** and **mode shapes** represent vibrational characteristics.

EOM for beam in bending:

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = 0$$

There are (theoretically) an infinite number of vibration modes.

Assume moe de shape to be:

$$W(x) = C_1 \cosh\frac{\lambda x}{L} + C_2 \sinh\frac{\lambda x}{L} + C_3 \cos\frac{\lambda x}{L} + C_4 \sin\frac{\lambda x}{L}$$

with $\lambda^4 = \omega^2 L^4 \frac{\rho A}{EI}$

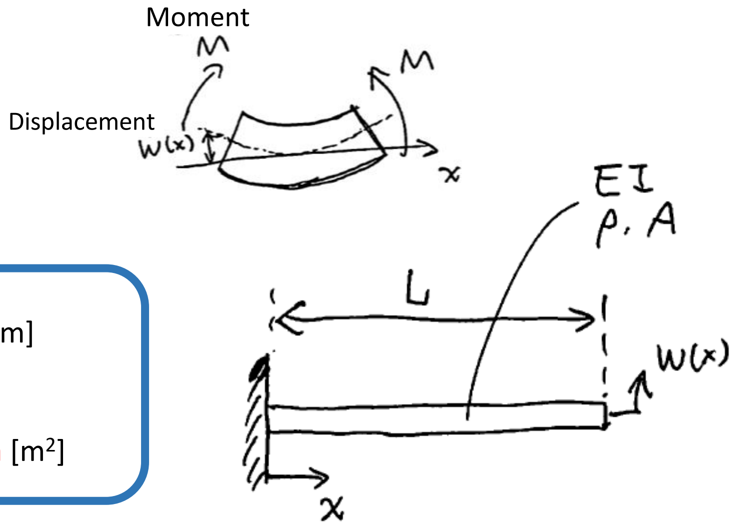
Natural frequencies are obtained by applying boundary conditions in EOM.

For a *cantilevered* beam with uniform cross-sectional area, **natural frequencies** and vibration **mode shapes** are as shown (right).

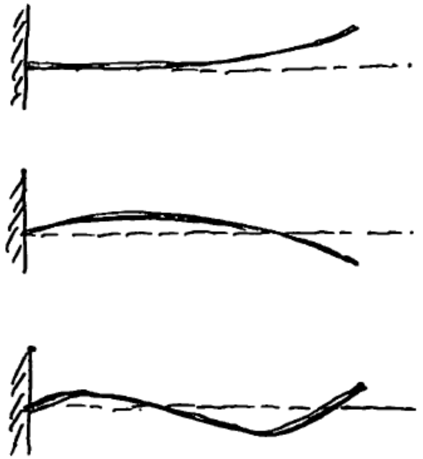
$w(x,t)$ Displacement [m]

ρ Density [kg/m^3]

A Cross-sectional area [m^2]



$$\begin{aligned} f_1 &= \frac{\Omega_1}{2\pi} = \frac{(1.875)^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \\ f_2 &= \frac{\Omega_2}{2\pi} = \frac{(4.694)^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \\ f_3 &= \frac{\Omega_3}{2\pi} = \frac{(7.855)^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \\ &\vdots \end{aligned}$$



1st mode

2nd mode

3rd mode

3. Theories for Structure Design

3.4 Finite element analysis (1/5)

[2] K. Komatsu, *Mechanical Structural Vibration*, Morikita Publishing, 2009. (in Japanese)

Consideration of an “infinite number of mode” is usually not necessary for satellite design. -> **Solid mechanics can be approximated by Finite-Element Method (FEM)**. (Ref. [2])

FEM gives approximate solution for partial differential equation. Displacements are interpolated by shape functions.

Beam example (this derivation is only to show FEM concept)

Static equation of beam (no inertial force): $EI\dfrac{\partial^4 w}{\partial x^4} = 0$

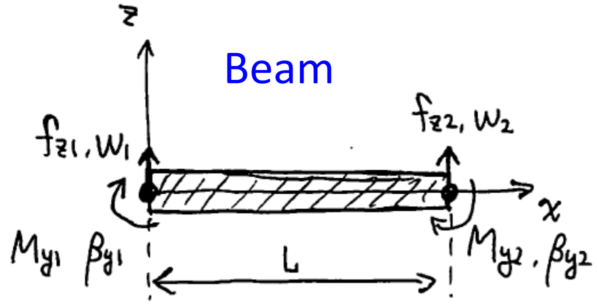
General solution of displacement is

$$w(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

Let’s consider the displacements and rotation angles at $x = 0, L$.

Displacement : $w(0),\ w(L)$

Rotation angle $\beta_y(0),\ \beta_y(L)$ and $\beta_y = -\dfrac{\partial w}{\partial x}$



From now on, the 2nd subscript shows node number (see figure above).

Displacement solution can be expressed by nodal displacements and rotation angles as

$$w(x) = \begin{bmatrix} 1-3\xi^2+2\xi^3 & L(-\xi+2\xi^2-\xi^3) & 3\xi^2-2\xi^3 & L(\xi^2-\xi^3) \end{bmatrix} \begin{bmatrix} w_{z1} \\ \beta_{y1} \\ w_{z2} \\ \beta_{y2} \end{bmatrix}$$

with $\xi = x/L$

This is expressed as $w(x) = \underline{\boldsymbol{N}(x)}\,\delta$

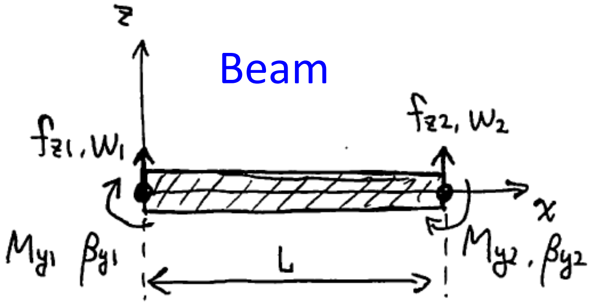
Shape function

i.e. Displacement at an arbitral location is expressed by displacement vector at two **nodes**.

3. Theories for Structure Design

3.4 Finite element analysis (2/5)

This simple beam example shows the concept of Finite-Element Method (FEM). (Ref. [2])



Substituting $w(x) = \boldsymbol{N}(x)\,\boldsymbol{\delta}$ into static force equation,

$$\begin{bmatrix} f_{z1} \\ M_{y1} \\ f_{z2} \\ M_{y2} \end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ 12 & -6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{bmatrix} w_{z1} \\ \beta_{y1} \\ w_{z2} \\ \beta_{y2} \end{bmatrix}$$

or $\boldsymbol{f} = \underline{\boldsymbol{K}}\,\boldsymbol{\delta}$

Stiffness matrix

Thus, if external forces and moments are given, displacements are solved.

Similarly, the mass matrix and damping matrix are obtained using the shape function.

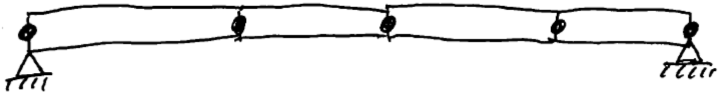
$$\boldsymbol{M}\ddot{\boldsymbol{\delta}} + \boldsymbol{C}\dot{\boldsymbol{\delta}} + \boldsymbol{K}\boldsymbol{\delta} = \boldsymbol{f}$$

➧ Dynamic responses are also calculated.

There are **2 modeling approaches** for rotational deformation.

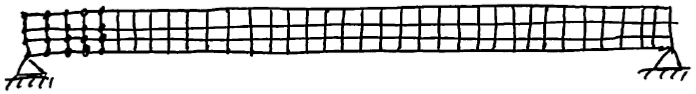
Using an **element with rotational DOF**, like a beam element.

✓ For 3D analysis, 1 node has 6 DOF.



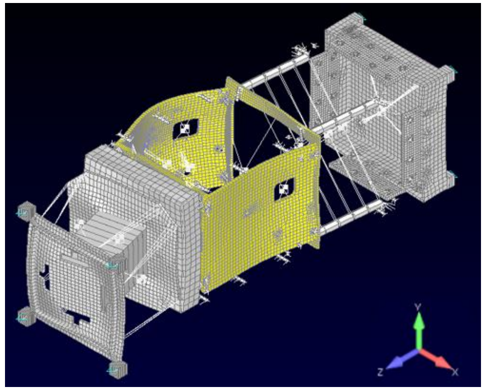
Using an **element without rotational DOF** (like plane stress element, solid element), and discretize in thickness direction as well.

✓ 1 node has 3 DOF.



3. Theories for Structure Design

3.4 Finite element analysis (3/5)



General analysis procedure of FEM (with commercial software)

1. Import the shape data of structure from 3D CAD.
2. Create **nodes** to divide into elements (= mesh generation)
3. Adjust shape, **elements**, and mesh according to analysis purpose.
   - ➢ Maintain aspect ratio of elements as low as possible.
   - ➢ Minimize number of DOF to reduce computational time (e.g. change to a simpler shape).
   - ➢ Use finer mesh where stress concentration occurs.
4. Input material properties (E, G, ν, I, A, etc.)
5. Set boundary conditions for displacement.
6. Set external forces (force boundary conditions).

7. Stiffness, mass, and damping matrices are generated for each element.
8. The matrices are assembled at shared nodes.
9. Boundary conditions are applied to the matrices.
10. The matrix equations are solved.
11. Stresses in each element are obtained from the obtained displacements.

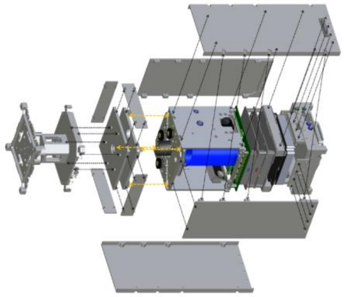
12. Visualization

Automatic

3. Theories for Structure Design

3.4 Finite element analysis (4/5)

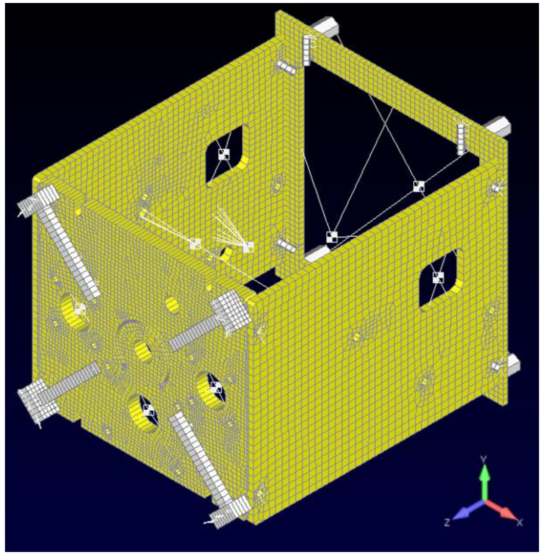
Example:
3U CubeSat
OrigamiSat-1
(2019 TokyoTech)

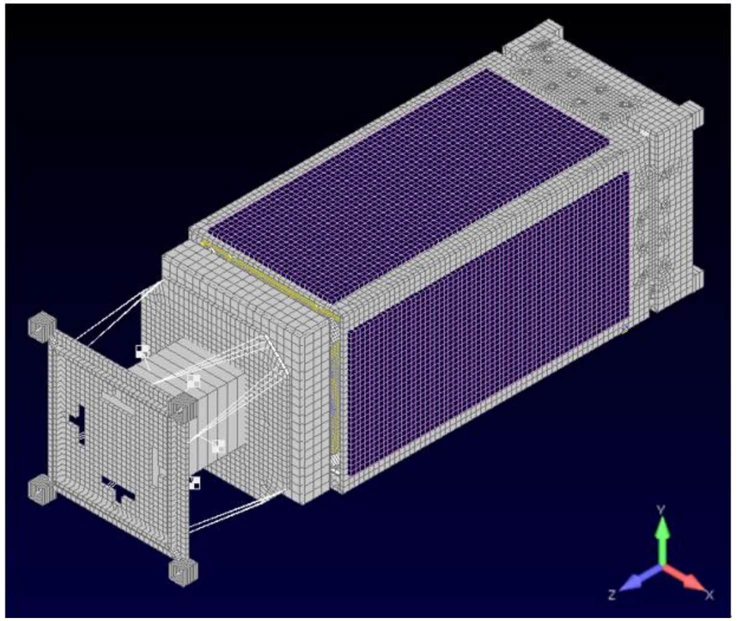


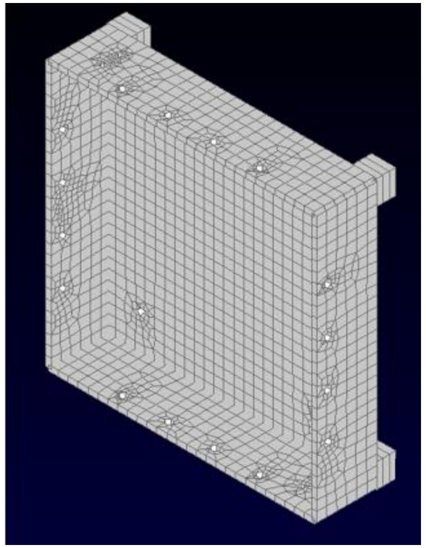
FEM analysis by Femap with FX Nastran 11.2.2.

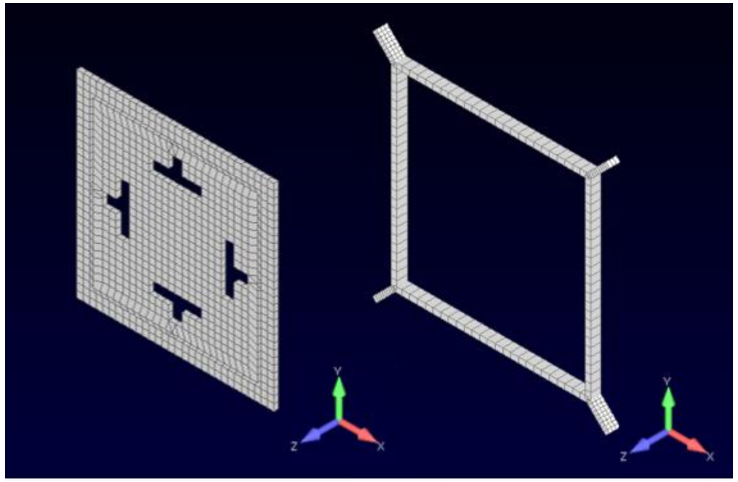
Models are simplified as follows.

- Basically, plate elements (with rotational DOF) are used instead of solid elements.
- Rigid-body elements are used for large components, and point-mass elements are used for small components.
- For bolts, beam elements are used.
  → Calculation is fast, but model generation is difficult.





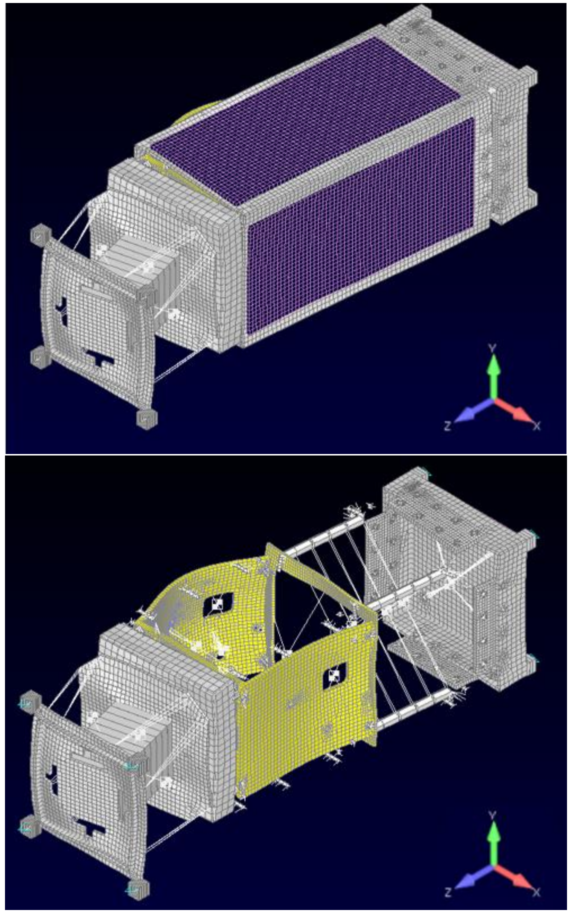




3. Theories for Structure Design

3.4 Finite element analysis (4/5)

Analysis results of
OrigamiSat-1
(2019 TokyoTech)



Structure design was verified by **two kinds of analysis**.

1. Modal vibration analysis

- 1st natural frequency is sufficiently high.
- Vibration mode shapes are reasonable.
- **After EM (STM) vibration test**, **Model Correlation** was conducted to match 1st natural frequency.

2. Stress analysis under constant acceleration

- First, calculated von Mises stress by applying acceleration with load factor (QT). (The sum of quasi-static acceleration + acceleration by sine vibration from rocket interface document.)
  - ⇨ Verify that the **Margin of Safety** is positive. (see Section 3.1)
    - ✓ Factor of safety: 1.25 for yield stress and 1.5 for ultimate stress
- **Next**, "random vibration" is converted to static acceleration by Miles equation (see SSP-52005) and stress analysis is conducted.
  - ⇨ Margin of Safety in all the members were confirmed to be positive.

Table: Predicted stresses and MS for yielding in bolts

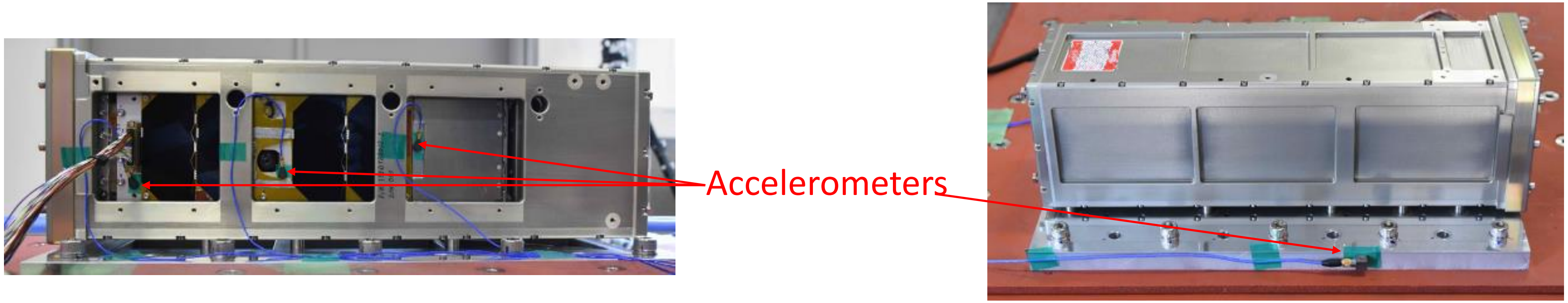
Bolt number	Maximum stress by external load [MPa]	Pre-stress [MPa]	Yielding stress of material [MPa]	**Margin of Safety for yielding**
#1 (M2.5, SUS304)	40.3	269	450	**0.455 > 0**
#2 (M3, SUS304)	54.2	272	450	**0.39 > 0**

4. Vibration and Shock Tests

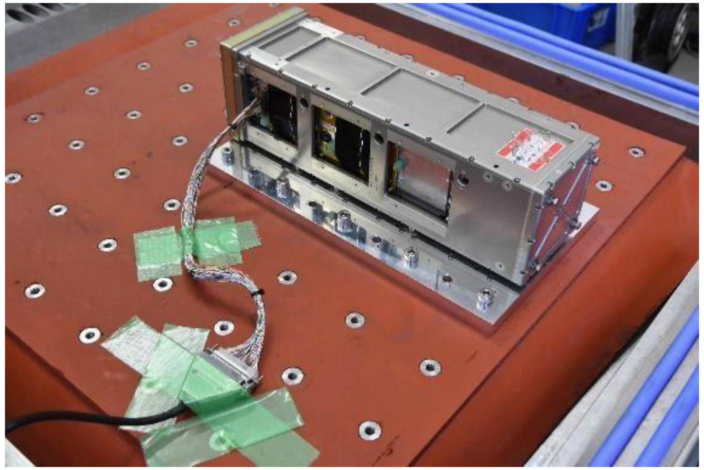
4. Vibration and Shock Tests

4.1 Vibration test (1/4)

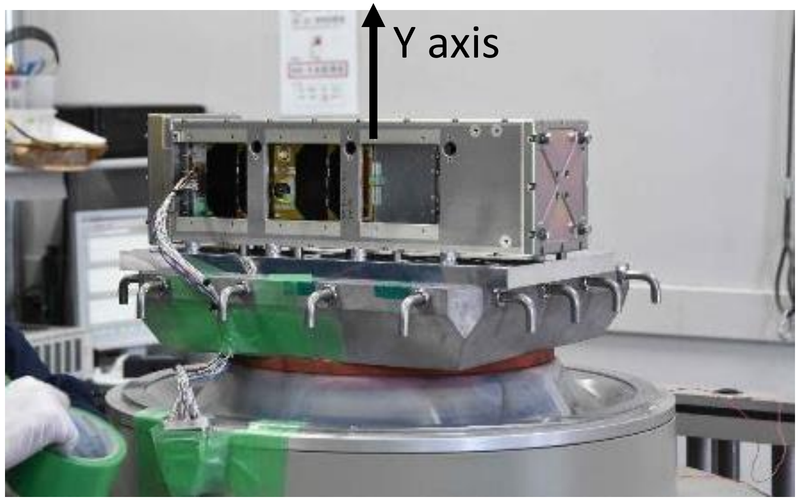
<u>Example</u>: <u>Acceptance Test</u> using FM of OrigamiSat-1



Horizontal direction (X, Z)



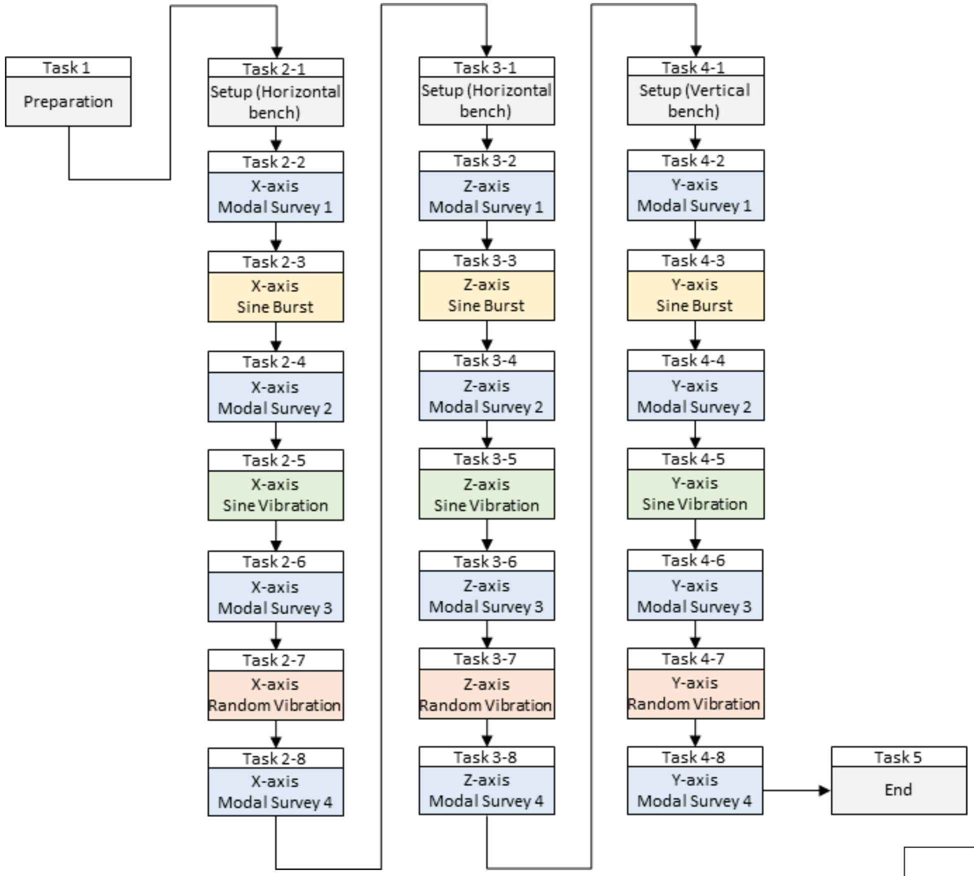
Vertical direction (Y)



4. Vibration and Shock Tests

4.1 Vibration test (2/4)

Test sequence: **4 kinds** of tests.



- Task 1: Preparation
- Task 2-1: Setup (Horizontal bench)
- Task 2-2: X-axis Modal Survey 1
- Task 2-3: X-axis Sine Burst
- Task 2-4: X-axis Modal Survey 2
- Task 2-5: X-axis Sine Vibration
- Task 2-6: X-axis Modal Survey 3
- Task 2-7: X-axis Random Vibration
- Task 2-8: X-axis Modal Survey 4
- Task 3-1: Setup (Horizontal bench)
- Task 3-2: Z-axis Modal Survey 1
- Task 3-3: Z-axis Sine Burst
- Task 3-4: Z-axis Modal Survey 2
- Task 3-5: Z-axis Sine Vibration
- Task 3-6: Z-axis Modal Survey 3
- Task 3-7: Z-axis Random Vibration
- Task 3-8: Z-axis Modal Survey 4
- Task 4-1: Setup (Vertical bench)
- Task 4-2: Y-axis Modal Survey 1
- Task 4-3: Y-axis Sine Burst
- Task 4-4: Y-axis Modal Survey 2
- Task 4-5: Y-axis Sine Vibration
- Task 4-6: Y-axis Modal Survey 3
- Task 4-7: Y-axis Random Vibration
- Task 4-8: Y-axis Modal Survey 4
- Task 5: End

(A) Modal Survey

Conducted before and after each test.

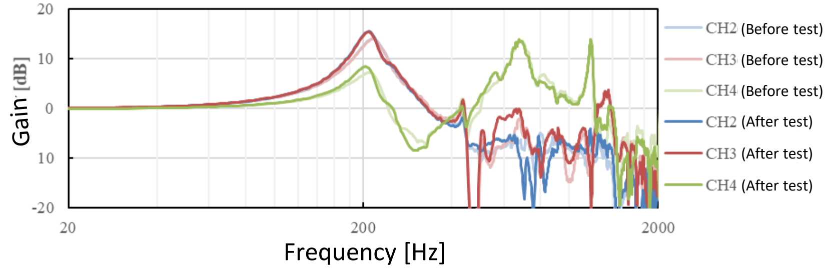
Evaluate the following using **low-level** random excitation (or sine sweep)

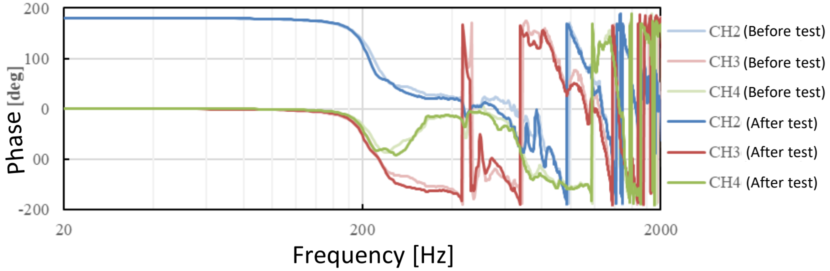
- ✓ Natural frequency
- ✓ Mode shape
- ✓ Mode damping

Notice any changes in natural frequency. → Indicates failure

Table Vibration condition for modal survey (low-level random excitation)

	Frequency [Hz]	Acceleration density [G^2/Hz]	RMS [Grms]	Duration [s]
X, Y, Z axis	20～2000	0.000127	0.5	60





4. Vibration and Shock Tests

4.1 Vibration test (3/4)

(B) Sine Burst

Quasi-static acceleration is simulated by a sine wave excitation with **a constant low frequency**.

Modal survey before and after the test to see any changes.

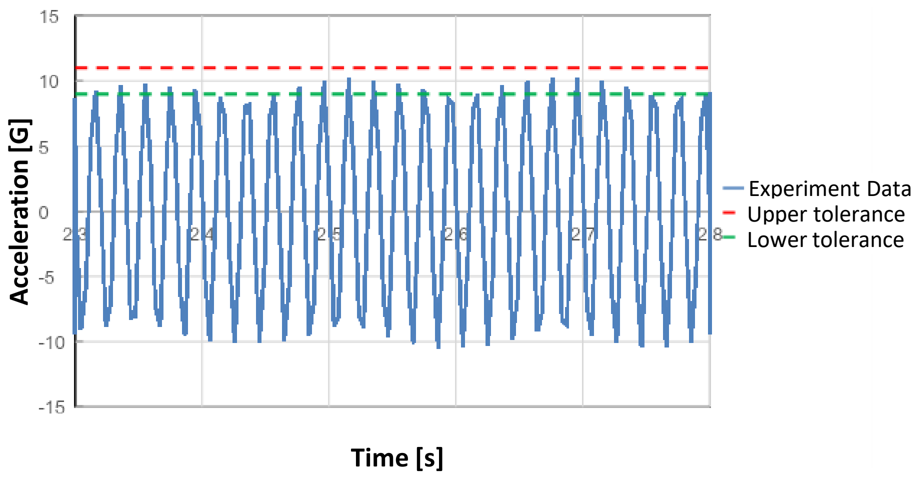


Figure Acceleration at monitor sensor in Y-axis sine burst test (AT)

(C) Sine Vibration

Sweep sine wave (gradually change excitation frequency) to simulate the excitation by rocket's vibration.

Modal survey before and after the test to see any changes.

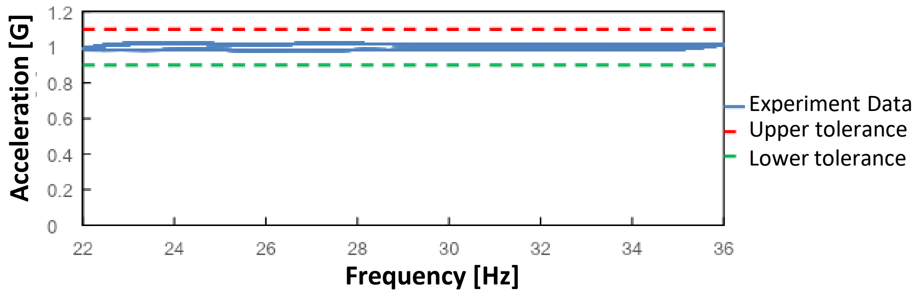


Figure Acceleration at monitor sensor in Y axis for sine sweep in 22-36Hz with 4 Oct/min

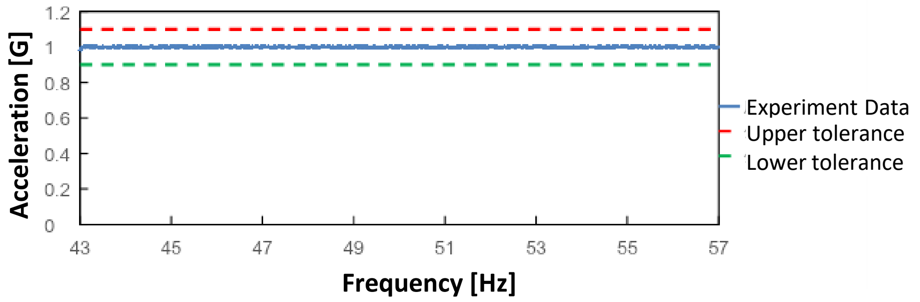


Figure Acceleration at monitor sensor in Y axis for sine sweep in 43-57Hz with 0.2 Oct/min

4. Vibration and Shock Tests

4.1 Vibration test (4/4)

(D) Random Vibration

Acoustic load is simulated by random vibration.

Modal survey before and after the test to see any changes.

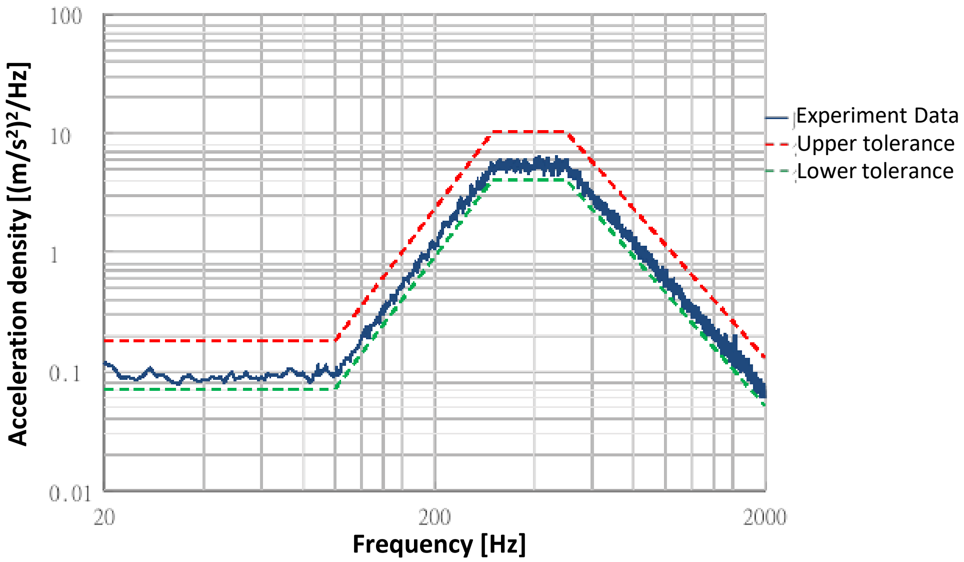


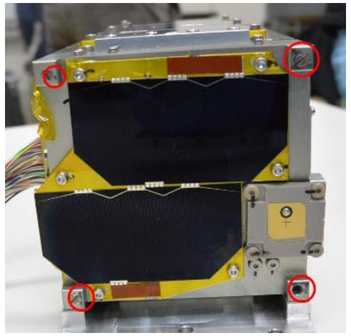
Figure Acceleration at monitor sensor in Y-axis random vibration test (AT)

Check list

福井試験場　12 月 25 日

No.	実施事項		確認項目	✓	作業者
1	持ち込チェック（目視）		チェックリストの物品を確認	✓	[redacted]
2	試験環境条件を記録する		温度・相対湿度を測定　16℃　52%　19℃　40%	✓	[redacted]
3	チャタリング検出装置動作確認		放出検知スイッチ ON/OFF により反転状態が表示されること	✓	[redacted]
4	バッテリの電圧を記録する		バッテリの電圧を計測する　7.98 V ✓	✓	〃
5	目視による外観検査を行う		変形、傷、汚れ等のないこと	✓	[redacted]
			[illegible]の保持解放機構に分離がないこと	✓	〃
			膜の保持解放機構のテグスに損傷、伸び等のないこと	✓	〃
			膜の保持解放機構のテグスの結び目にほつれがないこと	✓	〃
			展開アンテナのテグスに損傷、伸び等のないこと	✓	〃
			展開アンテナのテグスの結び目にほつれがないこと	✓	〃
			[illegible]カメラ部 HRM のマークにずれがないこと	✓	〃
			ネジ、ナット、テグス保護（ボビンフランジ）にトルクマークがあること。緩み、トルクマークのズレがないこと	✓	〃
			太陽電池セルに損傷・はがれがないこと	✓	〃
6	衛星全	POD の+Z 面を取り外す	締結トルク (M5: 4.01Nm、M6: 6.85Nm)		[redacted]

Inspection from outside

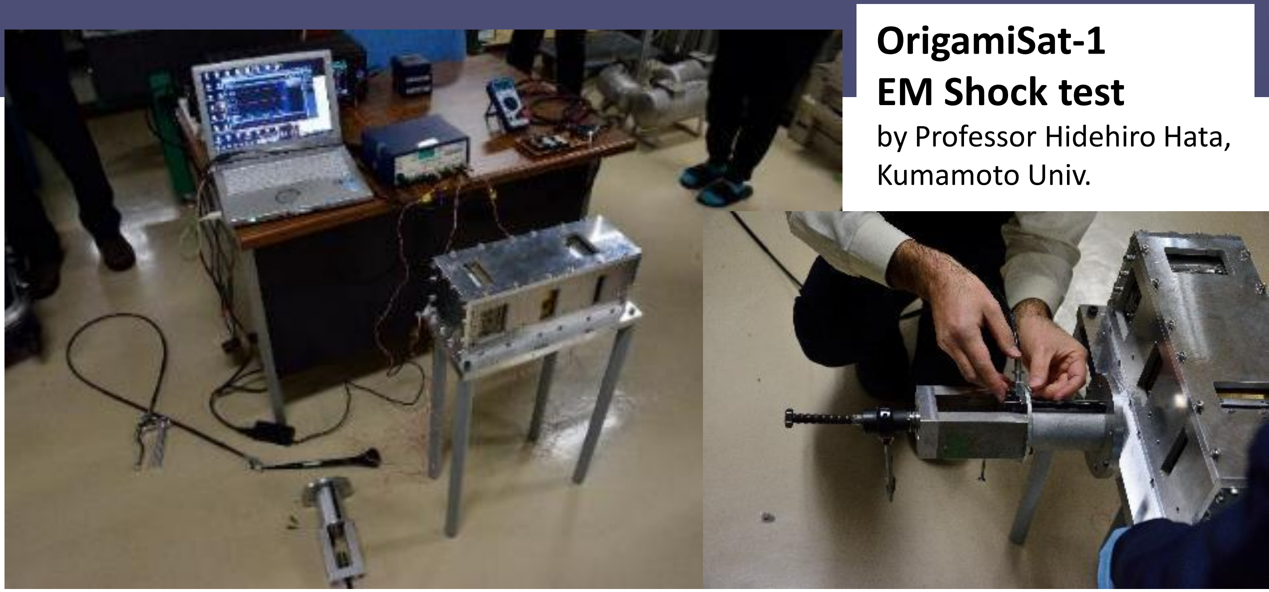


4. Vibration and Shock Tests

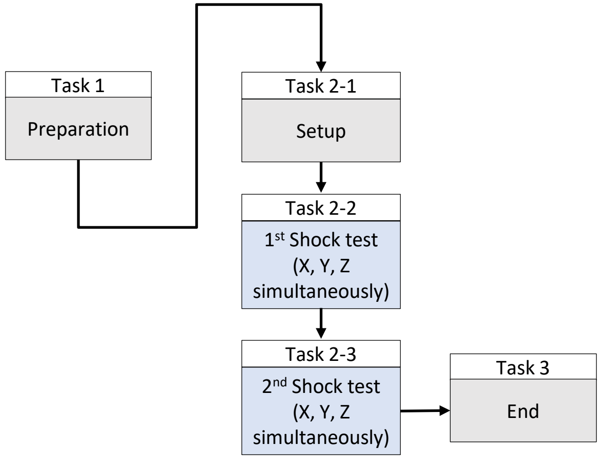
4.2 Shock test

<u>Example</u>: <u>Qualification Test</u> using EM of OrigamiSat-1

Shock condition is generally evaluated using **Shock Response Spectrum (SRS).**



OrigamiSat-1
EM Shock test
by Professor Hidehiro Hata,
Kumamoto Univ.



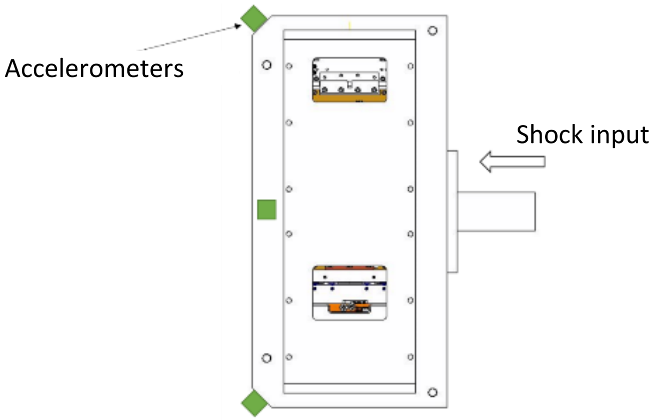
Task 1
Preparation

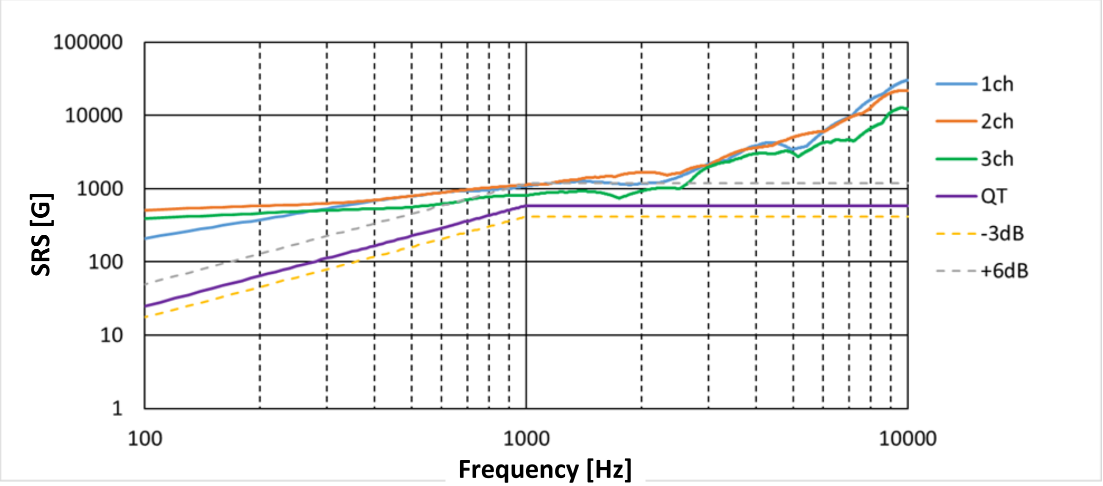
Task 2-1
Setup

Task 2-2
1st Shock test
(X, Y, Z
simultaneously)

Task 2-3
2nd Shock test
(X, Y, Z
simultaneously)

Task 3
End



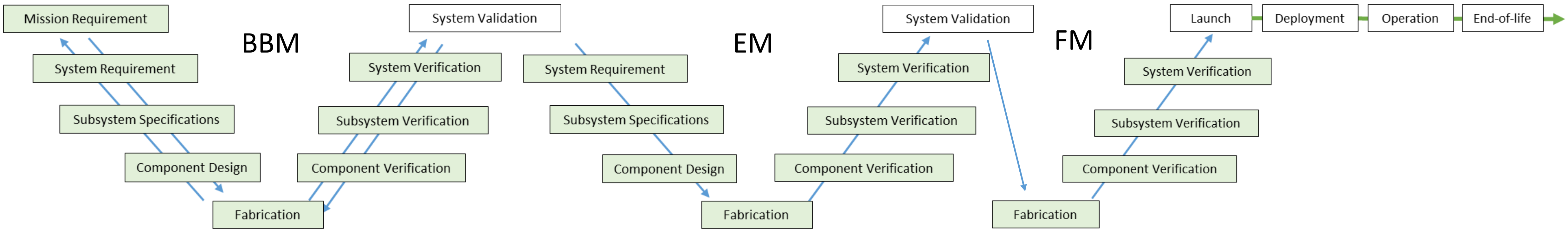


- ➢ QT-level shocks are applied more than 2 times.

7. Conclusion

7. Conclusion

Structure of a nano-satellite should be designed with careful consideration of the entire lifecycle of the system.


BBM

Mission Requirement

System Requirement

Subsystem Specifications

Component Design

Fabrication

Component Verification

Subsystem Verification

System Verification

System Validation

EM

System Requirement

Subsystem Specifications

Component Design

Fabrication

Component Verification

Subsystem Verification

System Verification

System Validation

FM

Fabrication

Component Verification

Subsystem Verification

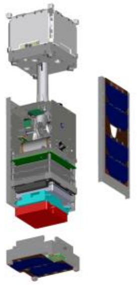
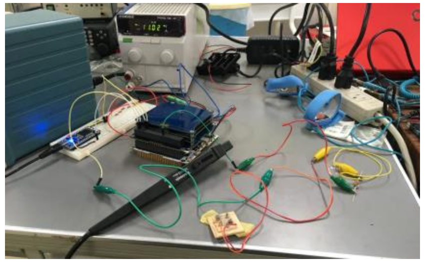
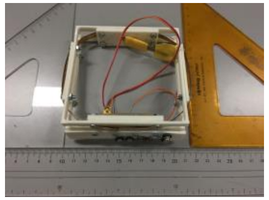
System Verification

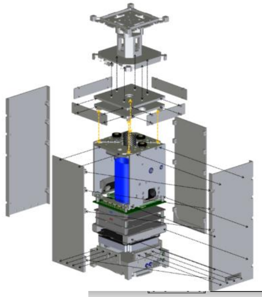
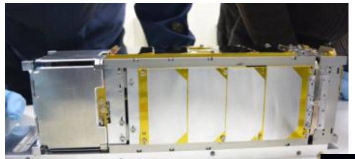
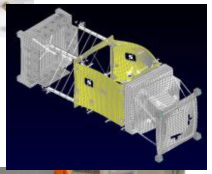
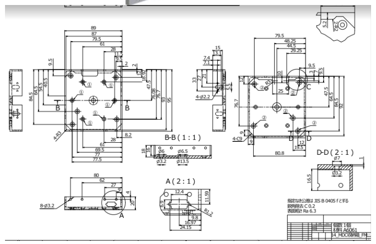
Launch

Deployment

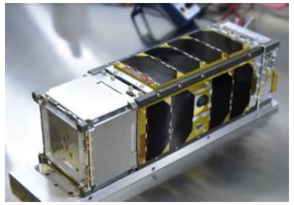
Operation

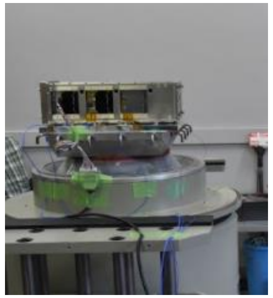
End-of-life




- ✓ CAD drawing
- ✓ Mock-up

- ✓ Machining drawing
- ✓ Structural analysis
- ✓ EM structure




- ✓ FM

Thank you very much.