GNSS terrestrial reference frames coincidence level since 2020

Speaker: Igor Gusev

The 17th meeting of the International Committee
on Global Navigation Satellite Systems
ICG-17

15–20 October 2023
Madrid, Spain

Contents

- **Part I:** Geodetic GNSS-infrastructure development in the Russian Federation
- **Part II:** The level of coincidence between terrestrial reference frames of GNSS

International Committee on
Global Navigation Satellite Systems

Geodetic GNSS-infrastructure development in the Russian Federation

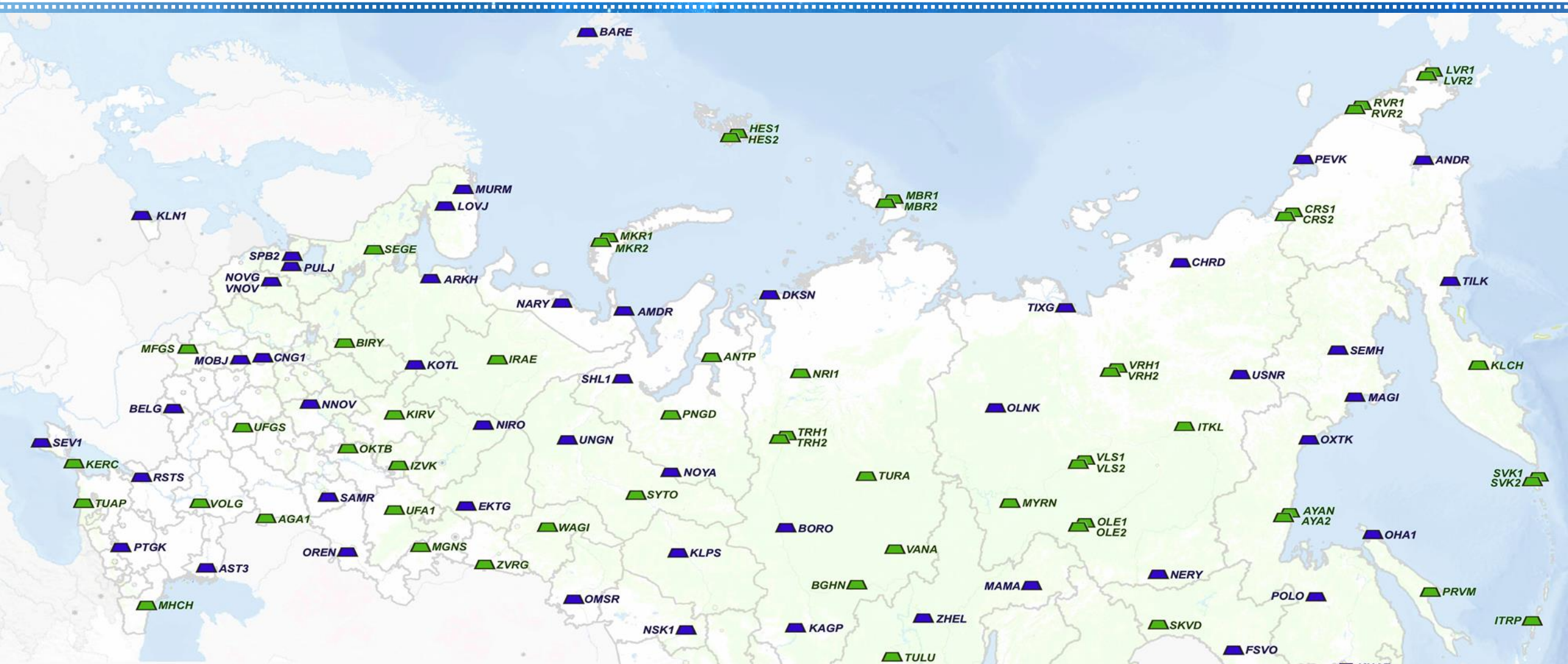
State reference frames in the Russian Federation

ROSCOSMOS

Parameter	GSK-2011	PZ-90.11
Purpose	For survey and mapping	For geodetic support of orbital missions and navigation
Coverage of the reference frame	Regional (National)	Global
Epoch	01.01.2011	01.01.2010
Geocentric gravitational constant (km^3/s^2) (including the atmosphere)	398 600,4415	398 600,4418
Nominal mean Earth's angular velocity (rad/s)	$7{,}292\ 115 \times 10^{-5}$	$7{,}292\ 115 \times 10^{-5}$
Equatorial radius of the Earth's ellipsoid (m)	6 378 136,5	6 378 136,0
Flattening factor of the Earth's ellipsoid	1/298,2564151	1/298,25784
Reference station network	Basic Astronomical and Geodetic Network (BAGN)	Space Geodetic Network (SGN) & Basic Astronomical and Geodetic Network (BAGN)*
The total number of reference stations in the Russian Federation	99	108

* Resolution of the Government of the Russian Federation from November 24, 2016 N 1240 "On the establishment of state reference frames, the state system of heights and the state gravimetric system"

Geodetic GNSS-Infrastructure Development



Legend:

– *BAGN stations (54)* ***installed*** *before January 1, 2022*

– *BAGN stations (33)* ***installed*** *after January 1, 2022*

– *dual BAGN stations (12)* ***installed*** *after January 1, 2022*

Examples of the installed BAGN stations equipped with GNSS receivers







Results

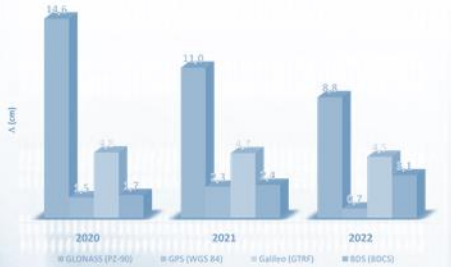
- By now the Basic Astronomical and Geodetic Network (BAGN) uniformly covers the entire territory of the Russian Federation. 99 BAGN stations have been installed and are operational in the Russian Federation, 12 of them are dual stations. Since 2022, the total number of BAGN stations has been doubled.

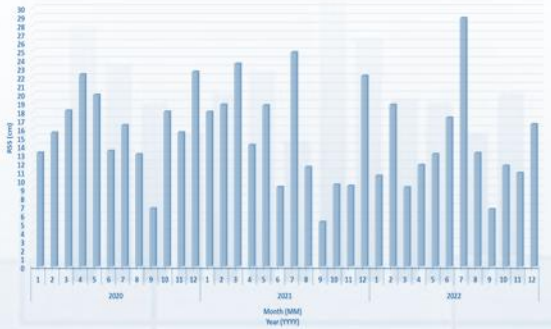
- The deployment of new BAGN stations was carried out in accordance with the technical requirements* of the International GNSS Service (IGS), thus they may be integrated into the IGS network.

- Due to the enhanced geodetic GNSS-infrastructure, the accuracy and stability of PZ-90.11 terrestrial reference frame will improve.

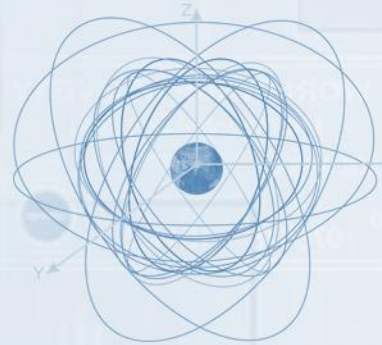
* IGS Site Guidelines. Infrastructure Committee Central Bureau. July 2015. URL: https://files.igs.org/pub/station/general/IGS%20Site%20Guidelines%20July%202015.pdf

ROSCOSMOS





The level of coincidence between terrestrial reference frames of GNSS



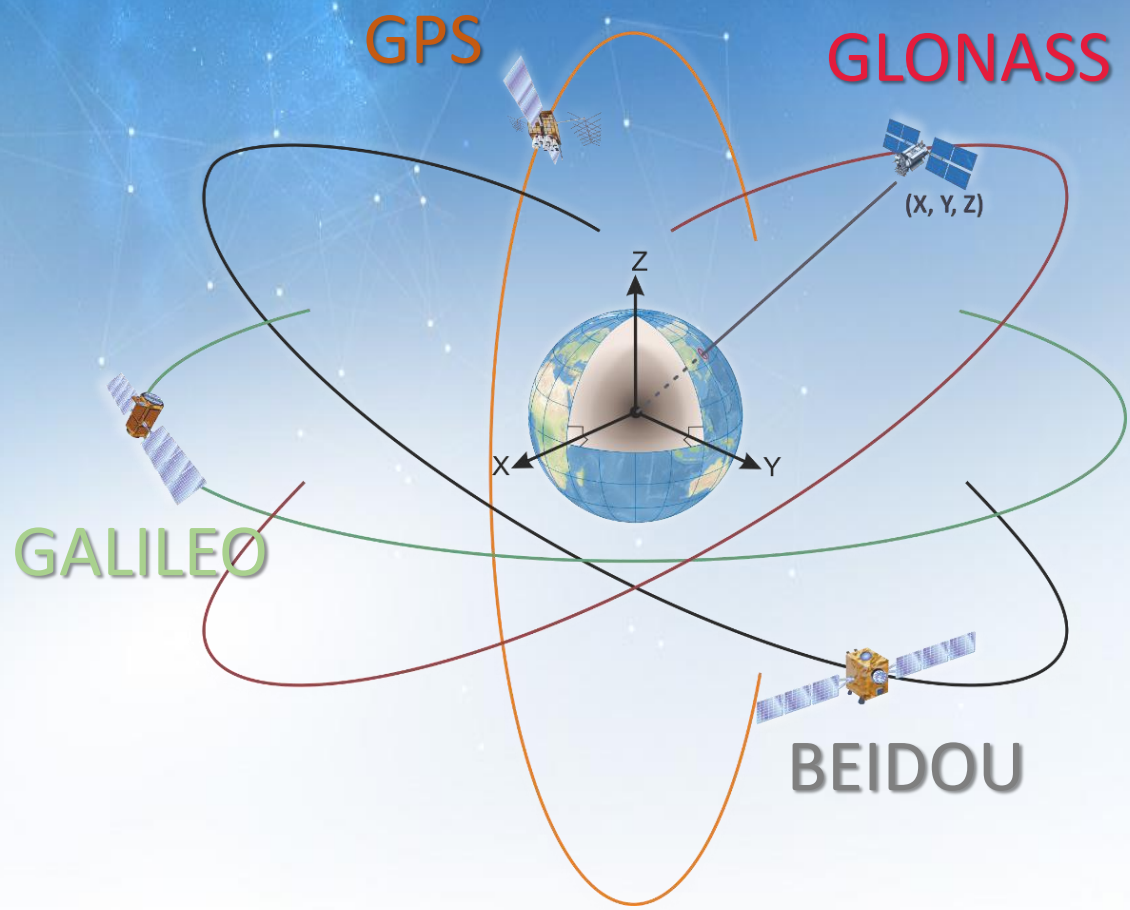
GNSS and their TRFs in broadcast messages

ROSCOSMOS

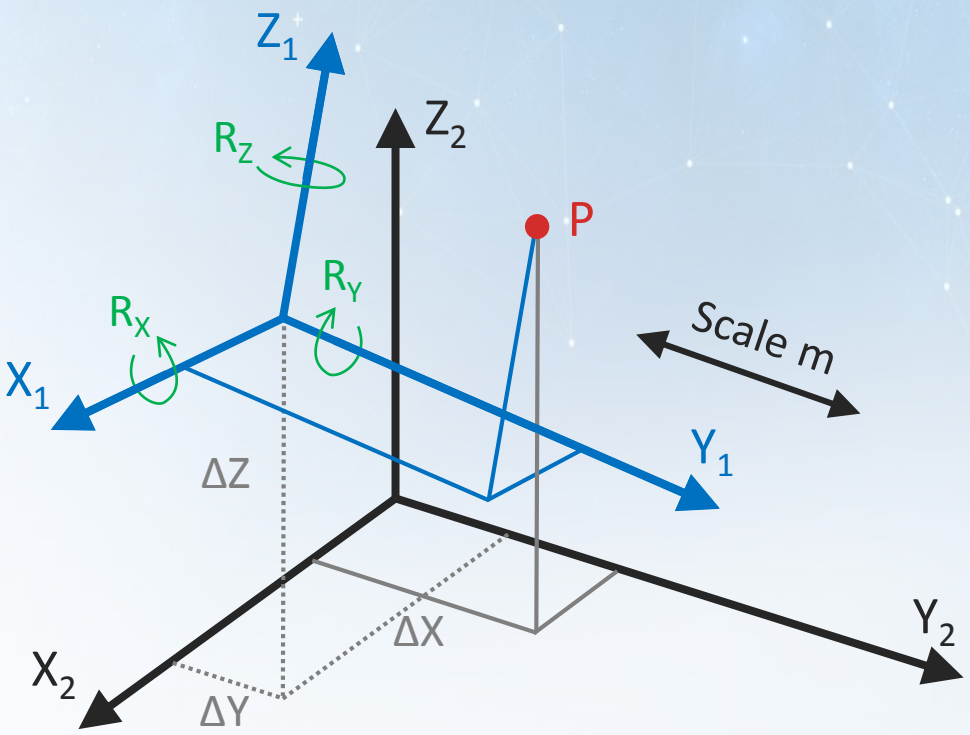
Terrestrial Reference Frames

#	GNSS	TRF	Reference Document	Web Link
1	**GPS**	**WGS 84**	Department of Defence World Geodetic System 1984: Its Definition and Relationships with Local Geodetic Systems. National Geospatial-Intelligence Agency (NGA) Standardization Document. v. 1.0.0. July 2014	https://nsgreg.nga.mil/doc/view?i=4085&month=9&day=4&year=2023
2	**GLONASS**	**PZ-90.11**	Specialized handbook “Parametry Zemli 1990” (PZ-90.11). Military Topographic Department of the General Staff of the Armed Forces of the Russian Federation. 2020	https://structure.mil.ru/files/pz-90.pdf
3	**Galileo**	**GTRF**	European GNSS (Galileo) Open Service – Service Definition Document. European Union Agency for the Space Programme. 2021	https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo-OS-SDD_v1.2.pdf
4	**BeiDou**	**BDCS**	BeiDou Coordinate System Template. China Satellite Navigation Office (CSNO). 2019 BeiDou Navigation Satellite System Signal In Space Interface Control Document. Open Service Signal B2b. China Satellite Navigation Office (CSNO). v. 1.0. 2020	http://en.beidou.gov.cn/SYSTEMS/Officialdocument/201912/P020191209570814877921.pdf http://en.beidou.gov.cn/SYSTEMS/Officialdocument/202008/P020200803544811195696.pdf

Global Navigation Satellite Systems



Two ways of coincidence level assessment



ΔX, ΔY, ΔZ – 3 parameters to specify the origin of the TRF

R_X, R_Y, R_Z – 3 rotation angles to specify the orientation of the TRF

m – scale parameter

Two ways of coincidence level assessment of GNSS terrestrial reference frames with International Terrestrial Reference Frame ITRF2020 were used in this study*:

✓ Annual coincidence

- Determination of the RSS_7 coincidence parameter

$$RSS_7 = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 + R_X^2 + R_Y^2 + R_Z^2 + m^2}$$

- Determination of the Λ coincidence parameter

$$\Lambda = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 + m^2 R_E^2 + (R_X^2 + R_Y^2 + R_Z^2)\frac{2R_E^2}{3}}$$

✓ Monthly repeatability

- Determination of the StD_{RSS_7} repeatability parameter

$$StD_{RSS_7} = \sqrt{\sigma_{\Delta X}^2 + \sigma_{\Delta Y}^2 + \sigma_{\Delta Z}^2 + \sigma_{R_X}^2 + \sigma_{R_Y}^2 + \sigma_{R_Z}^2 + \sigma_m^2}$$

- Determination of the StD_Λ repeatability parameter

$$StD_\Lambda = \sqrt{\sigma_{\Delta X}^2 + \sigma_{\Delta Y}^2 + \sigma_{\Delta Z}^2 + \sigma_m^2 R_E^2 + (\sigma_{R_X}^2 + \sigma_{R_Y}^2 + \sigma_{R_Z}^2)\frac{2R_E^2}{3}}$$

*S. Malys et al. (2021) Compatibility of Terrestrial Reference Frames used in GNSS broadcast messages during an 8 week period of 2019. Advances in Space Research 67 (2021) 834–844

Coincidence determination of TRFs: mathematical formalism

Annual coincidence

Daily Helmert parameters
$x_i = (\Delta X, \Delta Y, \Delta Z, R_X, R_Y, R_Z, m)_i$

Calculation of the standard deviation (StD) of the parameters over an annual interval:
$$\sigma = \sqrt{\frac{\sum_n (\bar{x} - x_i)^2}{n-1}}$$

Averaging over an annual interval:
$$\bar{x} = \frac{\sum_n x_i}{n}$$

Algorithm of the ***RSS$_7$*** calculation

Expression of angular elements and scale in a linear measure through the average radius of the Earth:
1 mas = 3.09 cm and 1 ppb = 0.64 cm with R_E = 6371 km

Determination of the ***RSS$_7$*** coincidence parameter:
$$RSS_7 = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 + R_X^2 + R_Y^2 + R_Z^2 + m^2}$$

Algorithm of the **Λ** calculation

Determination of the **Λ** coincidence parameter:
$$\Lambda = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 + m^2 R_E^2 + (R_X^2 + R_Y^2 + R_Z^2)\frac{2R_E^2}{3}}$$

TRF's repeatability determination: mathematical formalism

Monthly repeatability

Daily Helmert parameters

$$x_i = (\Delta X, \Delta Y, \Delta Z, R_X, R_Y, R_Z, m)_i$$

Averaging over a monthly interval:

$$\bar{x} = \frac{\sum_n x_i}{n}$$

Calculation of the parameters' StDs from their monthly sets:

$$\sigma = \sqrt{\frac{\sum_n (\bar{x} - x_i)^2}{n-1}}$$

Algorithm of the $\boldsymbol{StD_{RSS_7}}$ calculation

Expression of angular elements' and scale's StDs in a linear measure through the average radius of the Earth:
1 mas = 3.09 cm and 1 ppb = 0.64 cm with R_E = 6371 km

Determination of the $\boldsymbol{StD_{RSS_7}}$ repeatability parameter:

$$StD_{RSS_7} = \sqrt{\sigma^2_{\Delta X} + \sigma^2_{\Delta Y} + \sigma^2_{\Delta Z} + \sigma^2_{R_X} + \sigma^2_{R_Y} + \sigma^2_{R_Z} + \sigma^2_m}$$

Algorithm of the $\boldsymbol{StD_\Lambda}$ calculation

Determination of the $\boldsymbol{StD_\Lambda}$ repeatability parameter:

$$StD_\Lambda = \sqrt{\sigma^2_{\Delta X} + \sigma^2_{\Delta Y} + \sigma^2_{\Delta Z} + \sigma^2_m R_E^2 + (\sigma^2_{R_X} + \sigma^2_{R_Y} + \sigma^2_{R_Z})\frac{2R_E^2}{3}}$$

Estimated transformation parameters using GNSS broadcast messages
Data period: 01/01/2020 – 30/09/2023

ROSCOSMOS

TRF	Year	Number of solutions	Value	ΔX (m)	ΔY (m)	ΔZ (m)	$R_X \times 10^{-6}$ (rad)	$R_Y \times 10^{-6}$ (rad)	$R_Z \times 10^{-6}$ (rad)	Scale (ppb)	ΔX (cm)	ΔY (cm)	ΔZ (cm)	R_X (cm)	R_Y (cm)	R_Z (cm)	Scale (cm)	RSS_7 (cm)	Λ (cm)
WGS 84	2020	363	Mean	0,002	0,001	0,012	-0,001	0,000	0,000	-0,970	0,2	0,1	1,2	-0,7	-0,3	0,3	-0,6	**1,6**	**1,5**
			StD	0,008	0,008	0,038	0,004	0,005	0,007	0,438	0,8	0,8	3,8	2,3	3,1	4,2	0,3	6,9	6,1
	2021	364	Mean	0,001	-0,001	0,006	0,000	-0,001	-0,003	2,542	0,1	-0,1	0,6	-0,3	-0,5	-1,7	1,6	**2,5**	**2,3**
			StD	0,008	0,009	0,046	0,004	0,005	0,006	5,361	0,8	0,9	4,6	2,3	3,3	4,1	3,4	8,2	7,5
	2022	365	Mean	-0,001	-0,001	-0,001	0,000	-0,001	0,000	0,335	-0,1	-0,1	-0,1	-0,3	-0,7	-0,1	0,2	**0,8**	**0,7**
			StD	0,009	0,009	0,037	0,004	0,006	0,006	0,499	0,9	0,9	3,7	2,4	3,9	3,6	0,3	7,0	6,2
	2023	273	Mean	-0,001	-0,003	0,005	-0,001	0,000	-0,002	0,034	-0,1	-0,3	0,5	-0,5	0,0	-1,0	0,0	**1,2**	**1,1**
			StD	0,010	0,010	0,036	0,004	0,005	0,005	0,424	1,0	1,0	3,6	2,6	3,4	3,3	0,3	6,7	5,9
PZ-90	2020	363	Mean	0,001	0,003	0,144	-0,001	0,002	0,003	2,375	0,1	0,3	14,4	-0,4	1,4	1,9	1,5	**14,7**	**14,6**
			StD	0,015	0,014	0,056	0,005	0,005	0,018	0,725	1,5	1,4	5,6	2,9	3,3	11,2	0,5	13,4	11,5
	2021	364	Mean	0,001	0,002	0,106	0,000	0,001	0,005	2,468	0,1	0,2	10,6	0,0	0,4	3,1	1,6	**11,1**	**11,0**
			StD	0,015	0,015	0,069	0,006	0,006	0,029	0,845	1,5	1,5	6,9	3,5	4,0	18,4	0,5	20,5	17,2
	2022	365	Mean	0,000	-0,001	0,087	0,000	0,000	0,001	1,960	0,0	-0,1	8,7	0,0	0,3	0,5	1,2	**8,8**	**8,8**
			StD	0,017	0,016	0,054	0,006	0,007	0,033	0,970	1,7	1,6	5,4	3,7	4,4	21,1	0,6	22,7	18,8
	2023	273	Mean	0,001	-0,002	0,090	-0,001	0,001	0,002	1,438	0,1	-0,2	9,0	-0,4	0,7	1,0	0,9	**9,2**	**9,1**
			StD	0,018	0,015	0,052	0,005	0,007	0,031	1,222	1,8	1,5	5,2	3,5	4,5	19,9	0,8	21,5	17,9
GTRF	2020	363	Mean	-0,004	-0,001	-0,004	0,000	0,000	-0,003	7,113	-0,4	-0,1	-0,4	0,1	-0,3	-1,7	4,5	**4,9**	**4,8**
			StD	0,015	0,014	0,039	0,001	0,001	0,002	0,397	1,5	1,4	3,9	0,7	0,9	1,1	0,3	4,7	4,6
	2021	364	Mean	0,000	0,007	-0,006	0,000	0,000	-0,002	7,050	0,0	0,7	-0,6	-0,1	0,0	-1,4	4,5	**4,8**	**4,7**
			StD	0,014	0,015	0,039	0,001	0,001	0,002	0,386	1,4	1,5	3,9	0,8	0,9	1,1	0,2	4,7	4,6
	2022	365	Mean	-0,004	0,007	-0,004	0,000	0,000	0,000	6,865	-0,4	0,7	-0,4	-0,1	-0,2	-0,3	4,4	**4,5**	**4,5**
			StD	0,013	0,013	0,043	0,001	0,001	0,002	0,406	1,3	1,3	4,3	0,8	0,8	1,0	0,3	5,0	4,9
	2023	273	Mean	0,002	0,011	0,009	0,000	-0,001	-0,001	1,211	0,2	1,1	0,9	-0,3	-0,5	-0,8	0,8	**1,9**	**1,8**
			StD	0,011	0,011	0,041	0,001	0,001	0,002	3,565	1,1	1,1	4,1	0,7	0,9	1,0	2,3	5,2	5,1
BDCS	2020	365	Mean	0,001	0,001	0,000	0,001	0,003	0,001	-0,798	0,1	0,1	0,0	0,4	1,8	0,4	-0,5	**2,0**	**1,7**
			StD	0,007	0,006	0,050	0,006	0,010	0,011	0,380	0,7	0,6	5,0	3,8	6,3	7,3	0,2	11,5	9,9
	2021	364	Mean	0,005	0,000	-0,004	0,002	-0,002	0,003	-0,890	0,5	0,0	-0,4	1,2	-1,5	2,0	-0,6	**2,9**	**2,4**
			StD	0,007	0,006	0,048	0,007	0,011	0,011	0,340	0,7	0,6	4,8	4,4	6,7	7,0	0,2	11,7	10,0
	2022	364	Mean	0,005	-0,002	-0,004	0,003	-0,004	0,003	-0,935	0,5	-0,2	-0,4	2,1	-2,3	1,9	-0,6	**3,8**	**3,1**
			StD	0,006	0,006	0,041	0,007	0,014	0,012	0,370	0,6	0,6	4,1	4,5	9,0	7,8	0,2	13,4	11,2
	2023	269	Mean	0,003	0,004	0,012	0,001	0,003	-0,003	-1,020	0,3	0,4	1,2	0,7	2,1	-1,8	-0,6	**3,2**	**2,7**
			StD	0,011	0,014	0,032	0,008	0,014	0,011	0,546	1,1	1,4	3,2	5,0	8,7	6,8	0,3	12,7	10,6

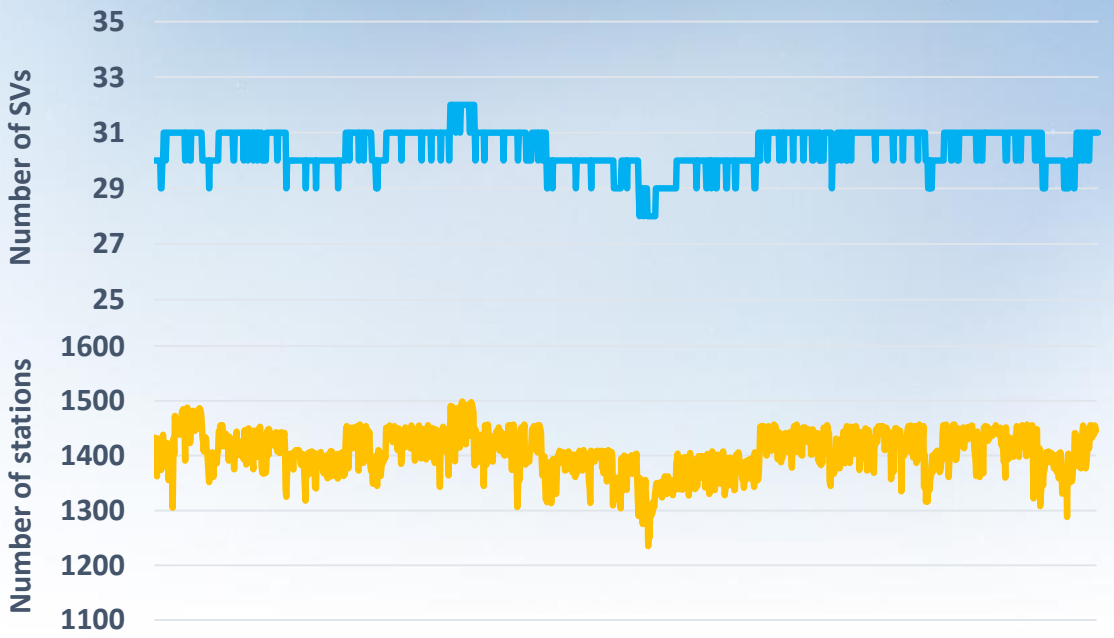
Coincidence of PZ-90.11 orbital realization and ITRF2020

PZ-90.11	2020	2021	2022	2023
Mean (cm)	14,7	11,1	8,8	9,2
StD (cm)	13,4	20,5	22,7	21,5

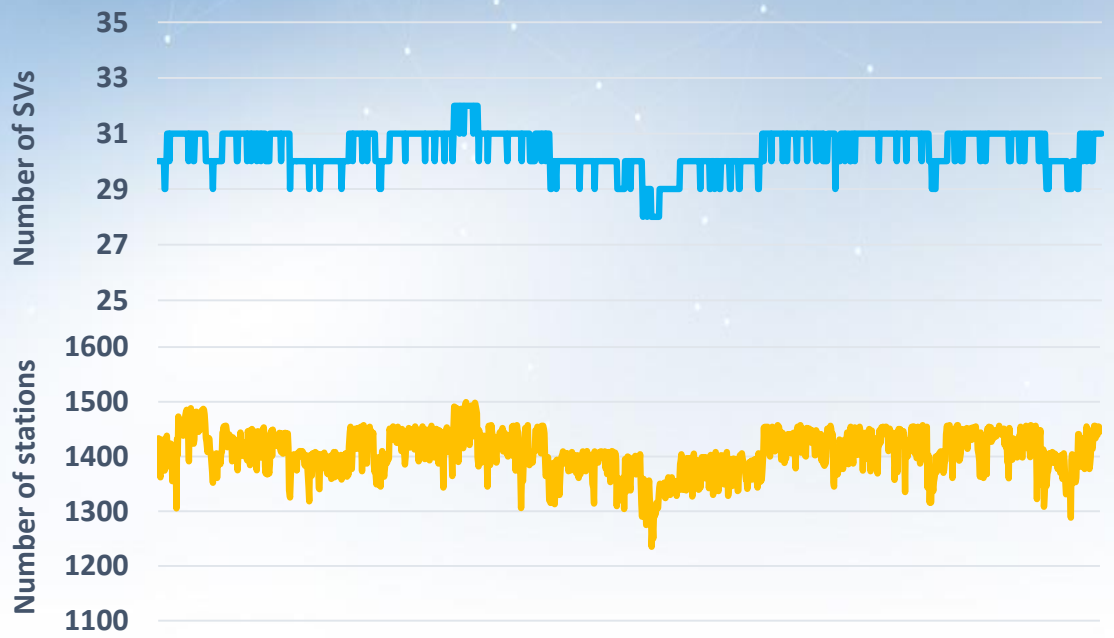
PZ-90.11	2020	2021	2022	2023
Mean (cm)	14,6	11,0	8,8	9,1
StD (cm)	11,5	17,2	18,8	17,9

Coincidence of WGS 84 orbital realization and ITRF2020

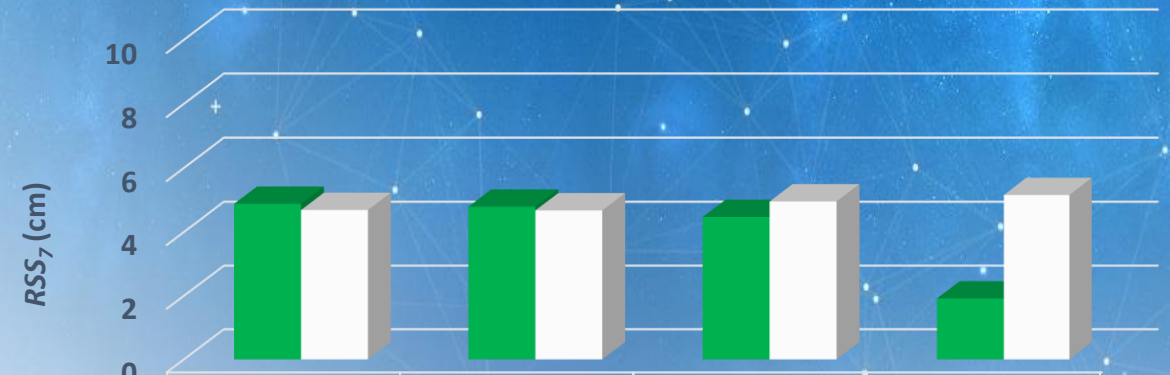
	2020	2021	2022	2023
Mean (cm)	1,6	2,5	0,8	1,2
StD (cm)	6,9	8,2	7,0	6,7



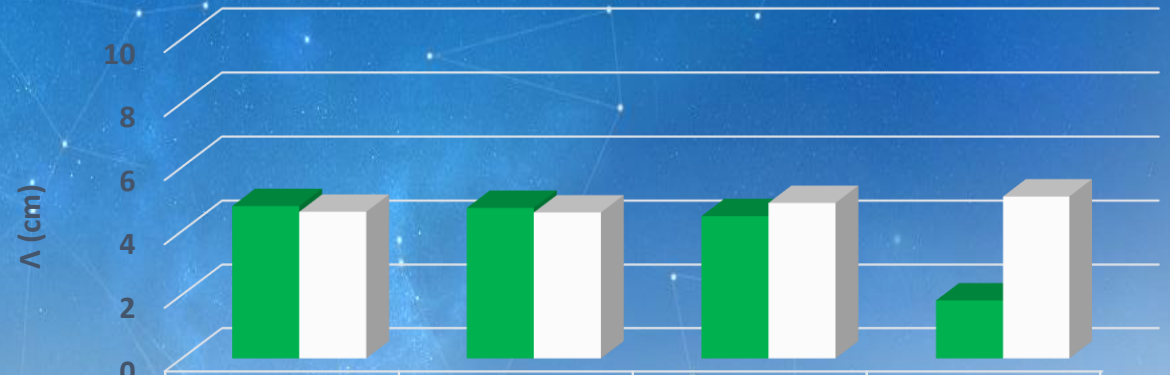
	2020	2021	2022	2023
Mean (cm)	1,5	2,3	0,7	1,1
StD (cm)	6,1	7,5	6,2	5,9



Coincidence of GTRF orbital realization with ITRF2020

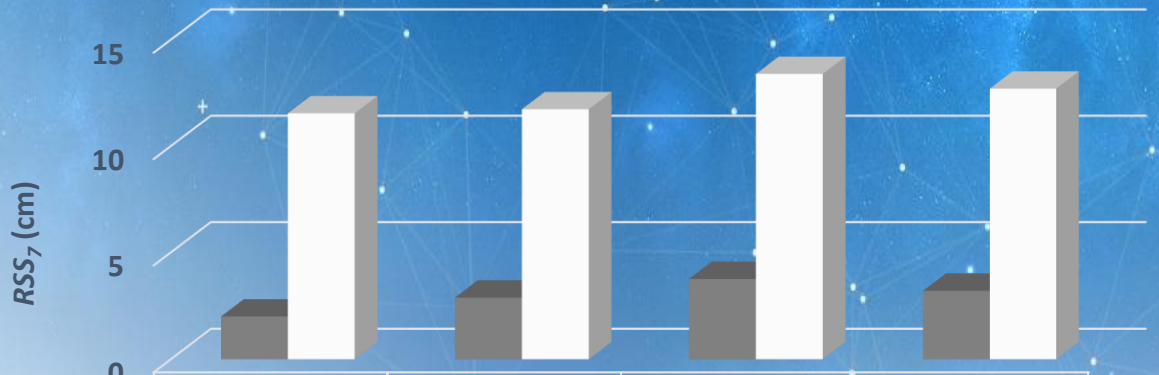


	2020 GTRF20	2021 GTRF21	2022 GTRF22	2023 GTRF23
Mean (cm)	4,9	4,8	4,5	1,9
StD (cm)	4,7	4,7	5,0	5,2

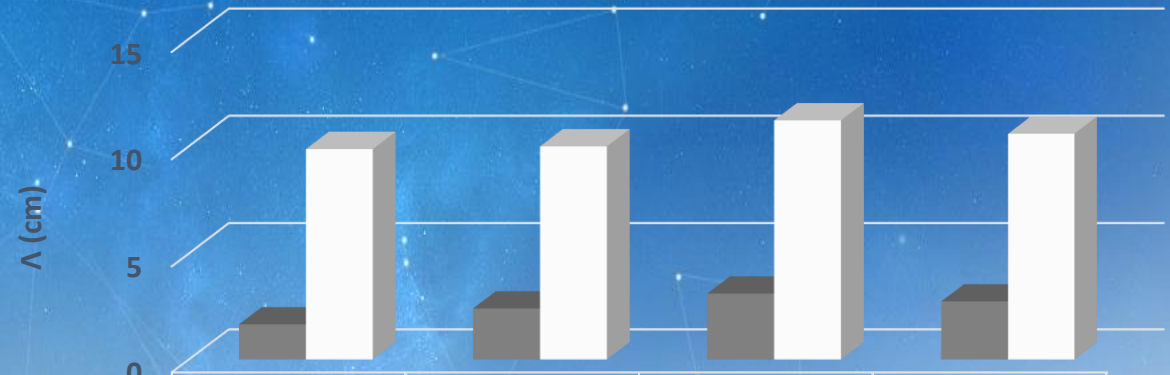


	2020 GTRF20	2021 GTRF21	2022 GTRF22	2023 GTRF23
Mean (cm)	4,8	4,7	4,5	1,8
StD (cm)	4,6	4,6	4,9	5,1

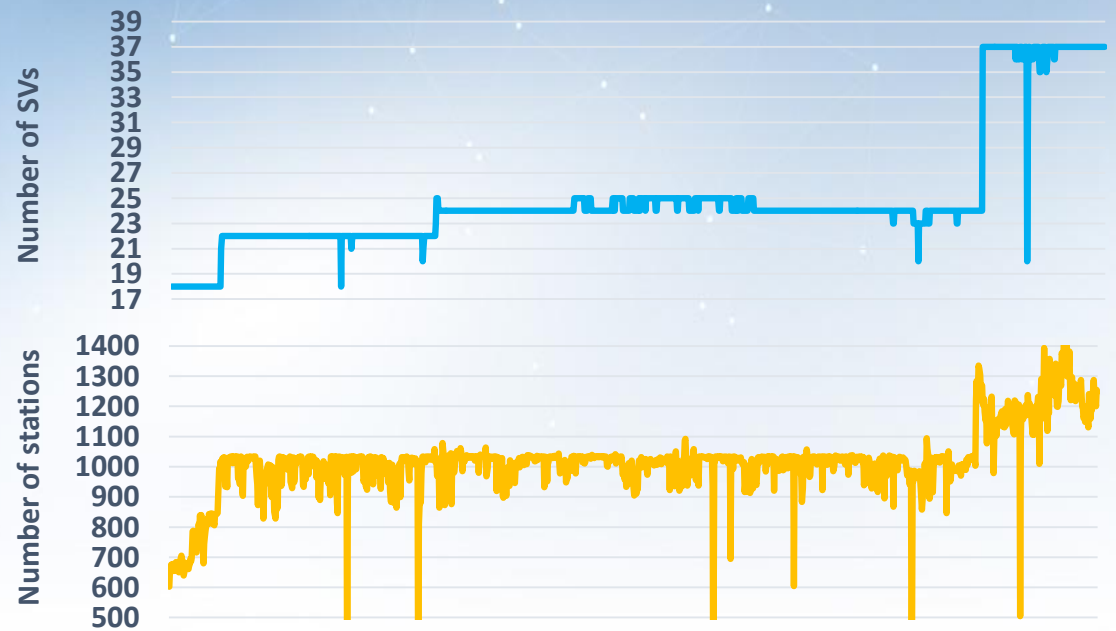
Coincidence of BDCS orbital realization with ITRF2020



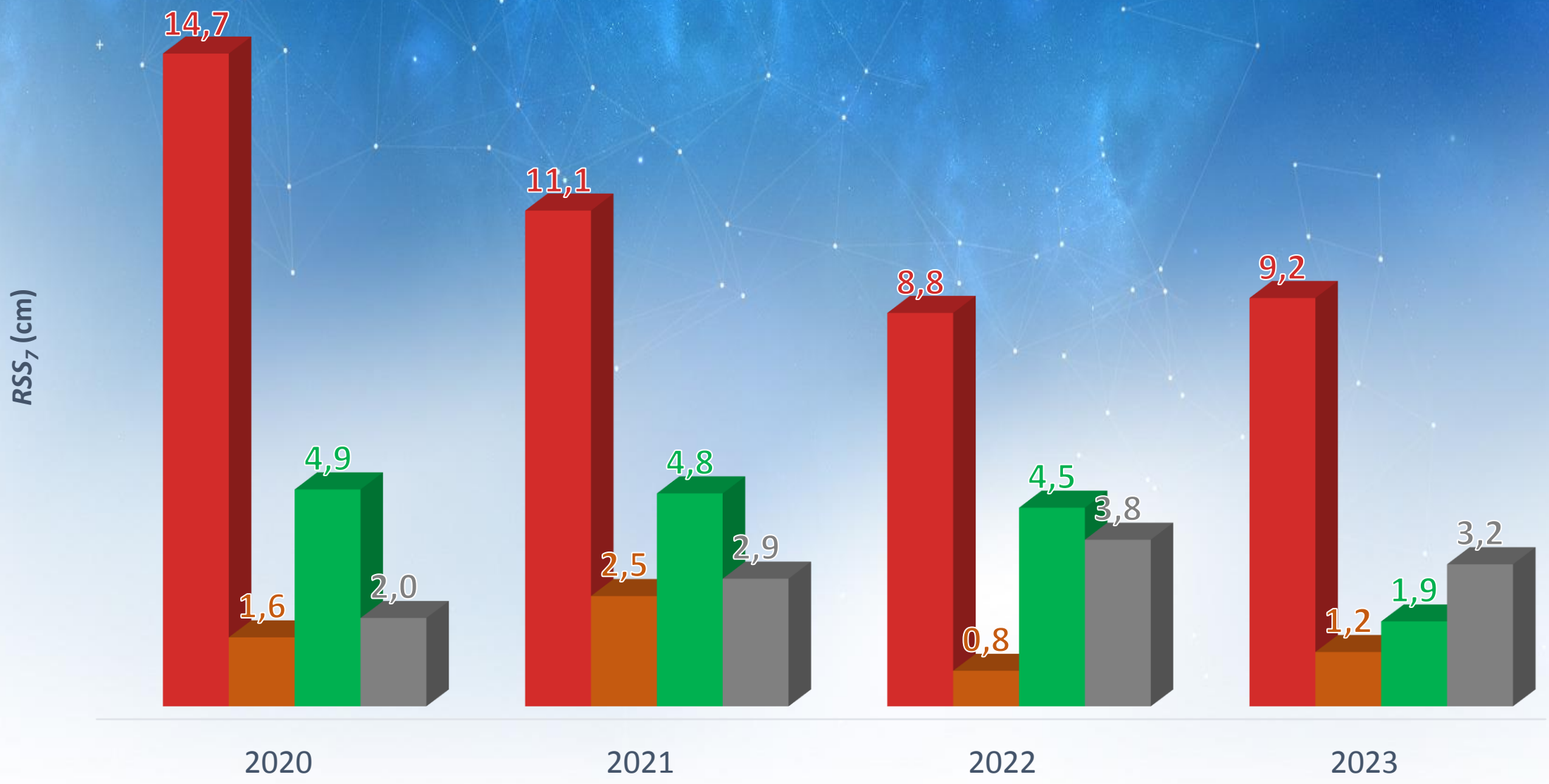
	2020	2021	2022	2023
Mean (cm)	2,0	2,9	3,8	3,2
StD (cm)	11,5	11,7	13,4	12,7



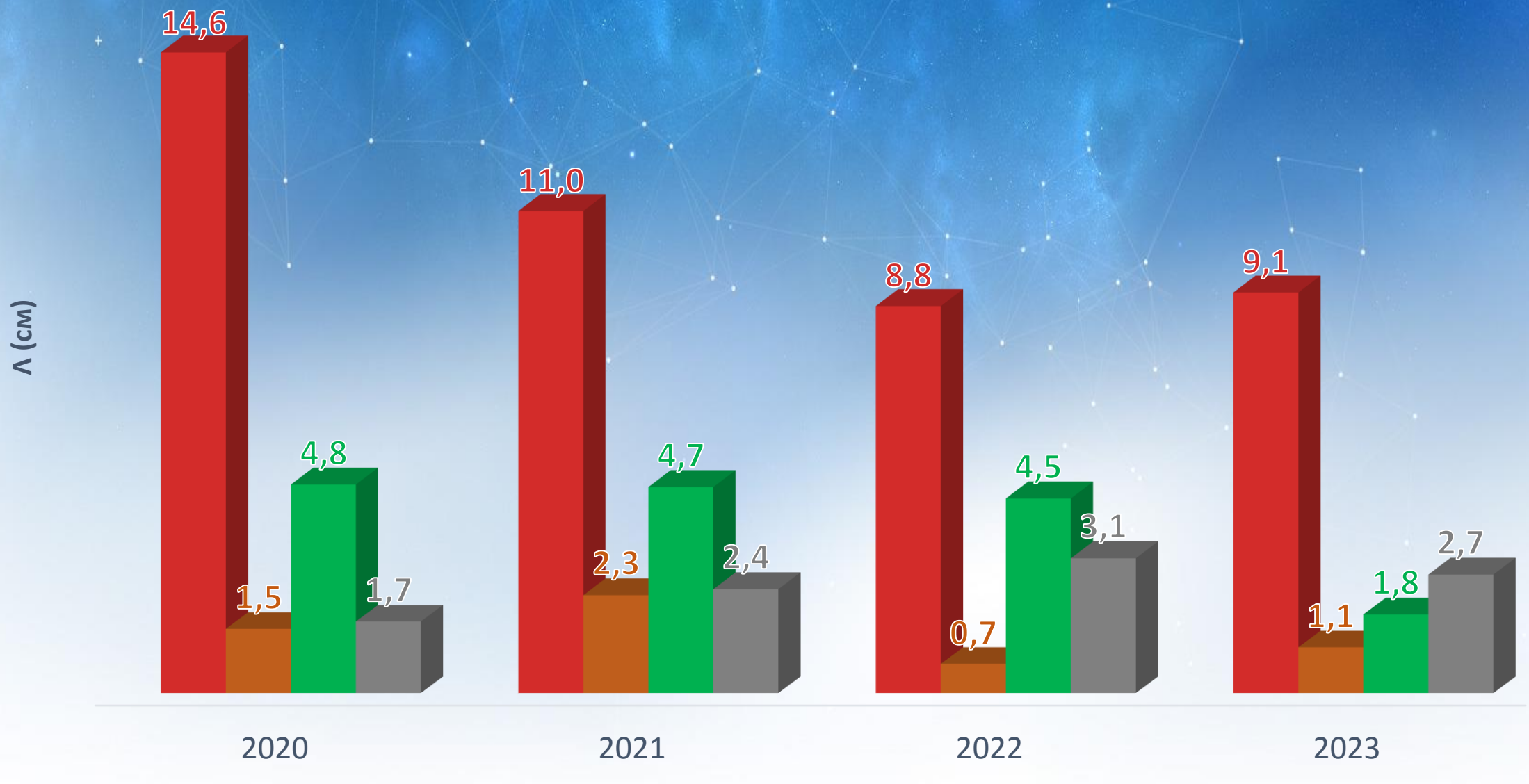
	2020	2021	2022	2023
Mean (cm)	1,7	2,4	3,1	2,7
StD (cm)	9,9	10,0	11,2	10,6



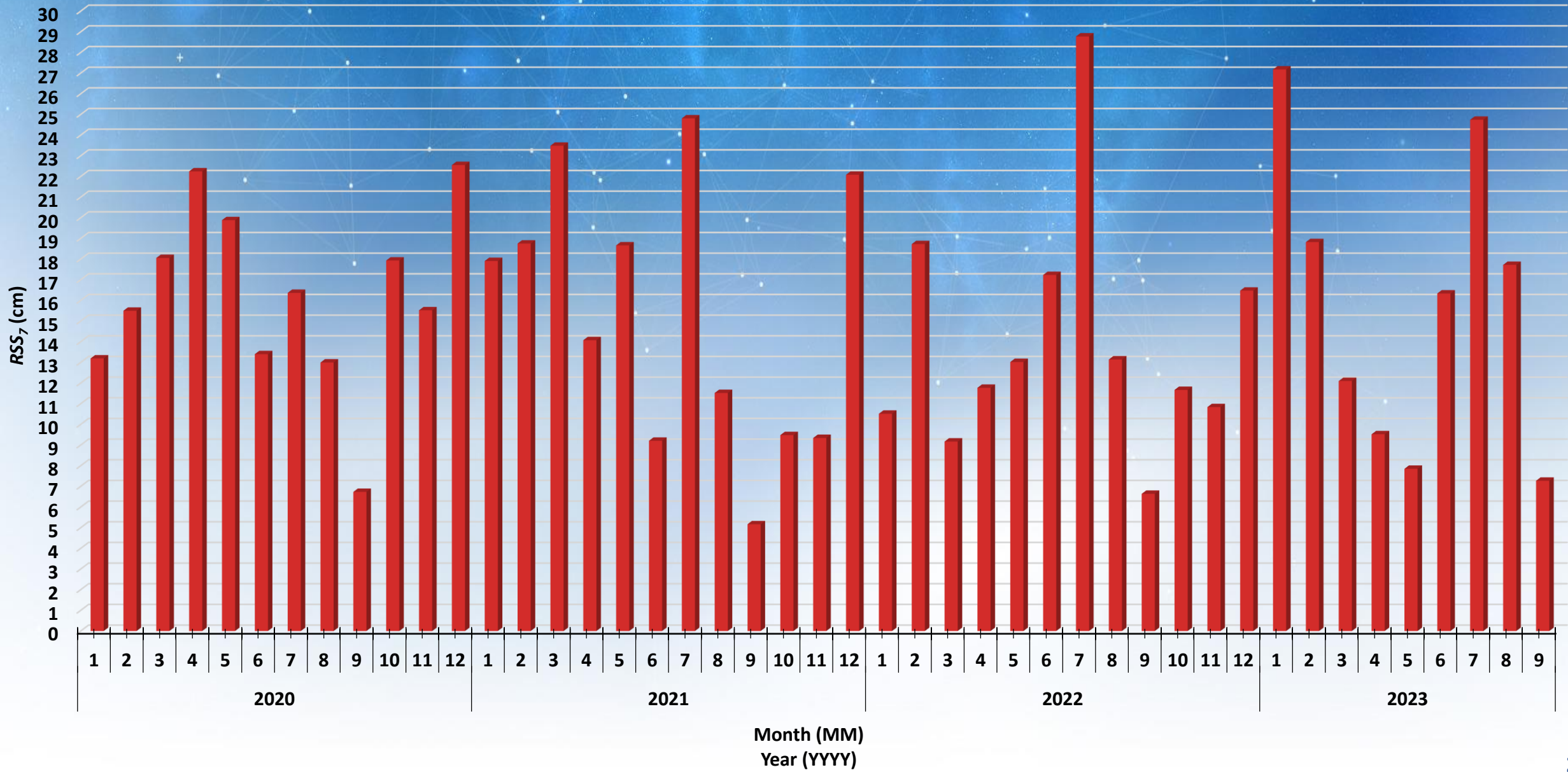
Coincidence in terms of RSS_7 between GNSS TRFs and ITRF2020 during 2020-2023



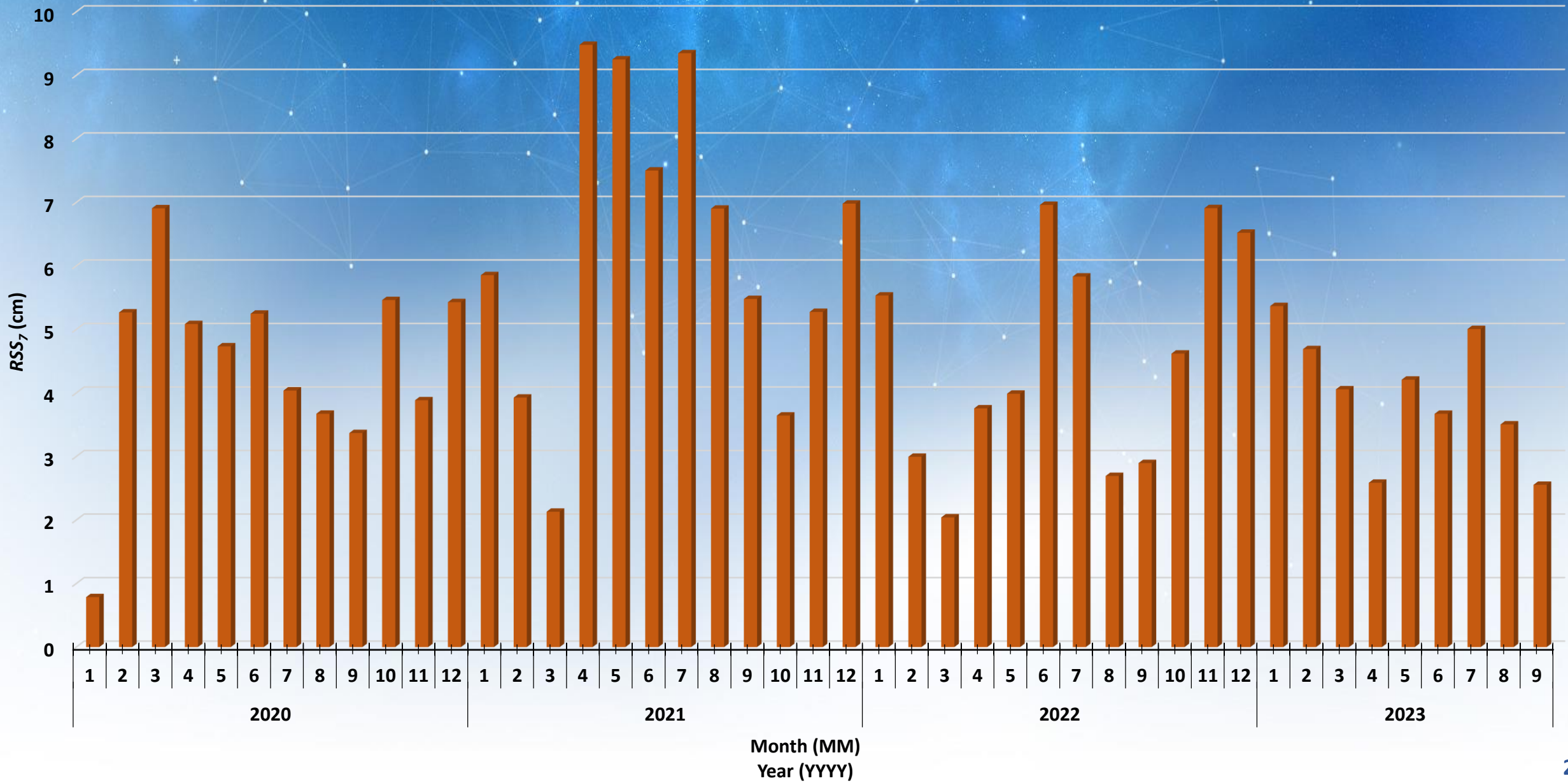
Coincidence in terms of Λ between GNSS TRFs and ITRF2020 during 2020-2023



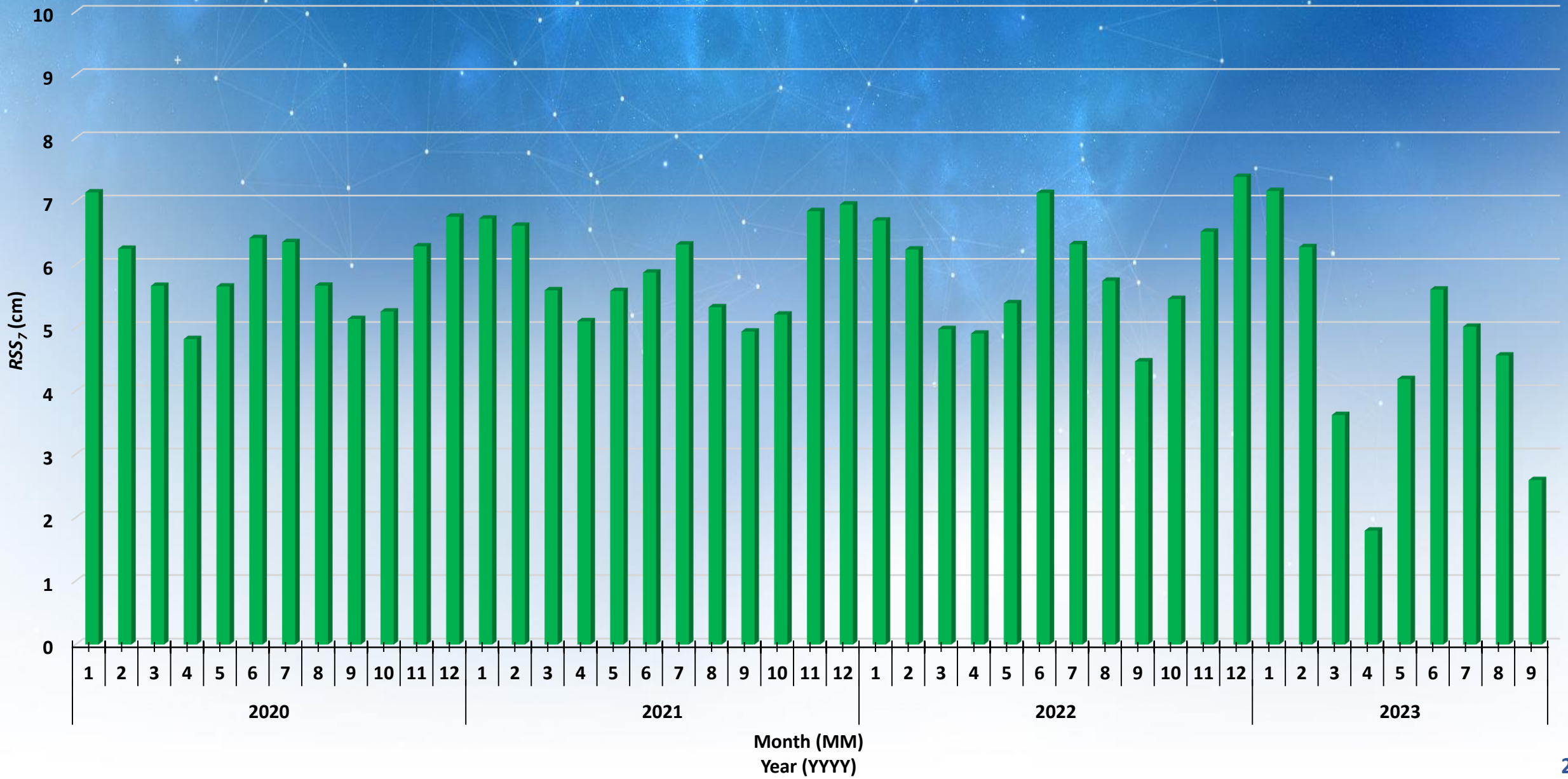
Monthly repeatability of the GLONASS transformation parameters in terms of RSS_7



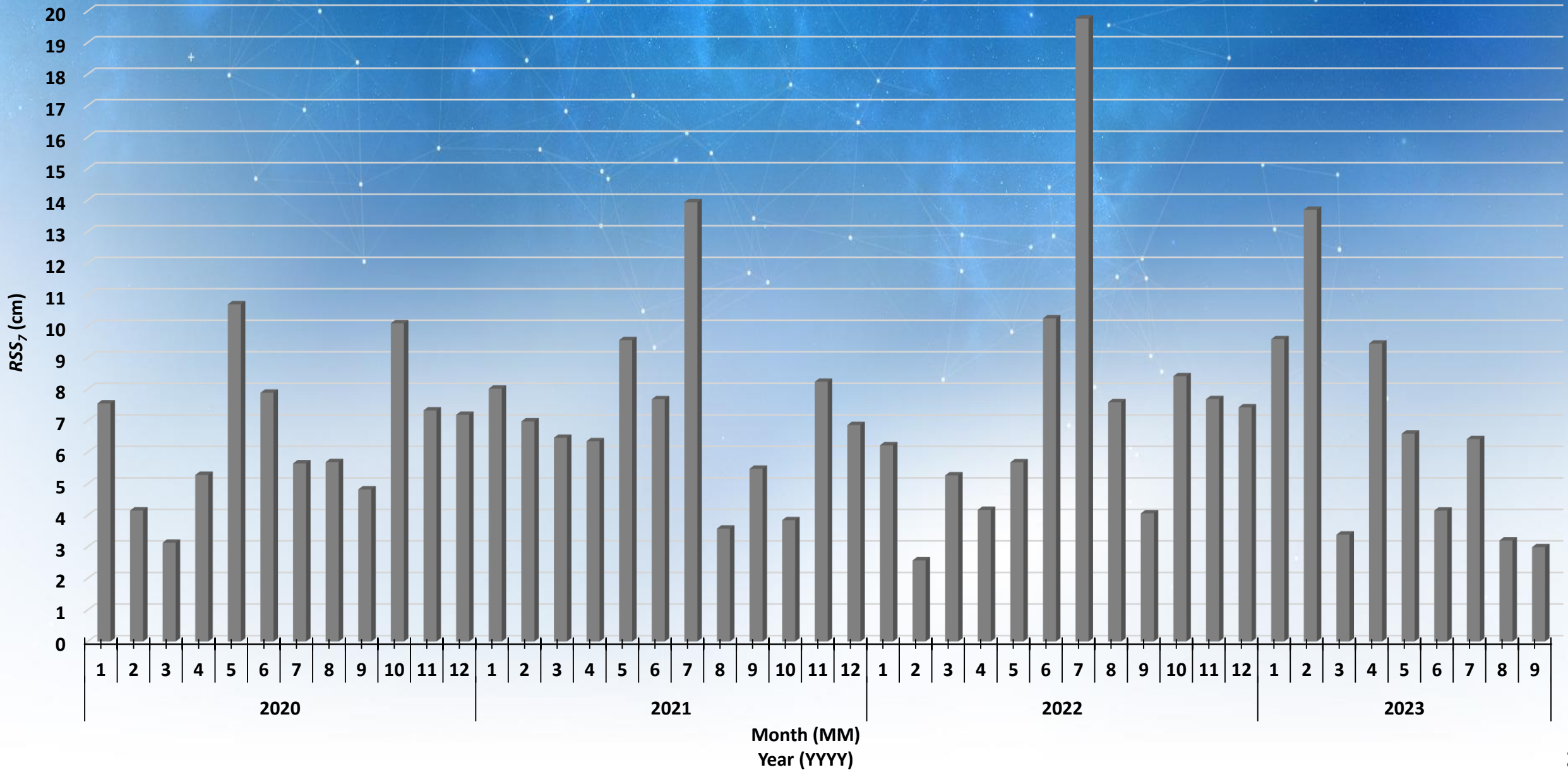
Monthly repeatability of the GPS transformation parameters in terms of RSS_7



Monthly repeatability of the Galileo transformation parameters in terms of RSS_7



Monthly repeatability of the BDS transformation parameters in terms of RSS_7



Standard deviations of the monthly sets of transformation parameters and the associated StD_{RSS_7} and StD_{Λ} values*

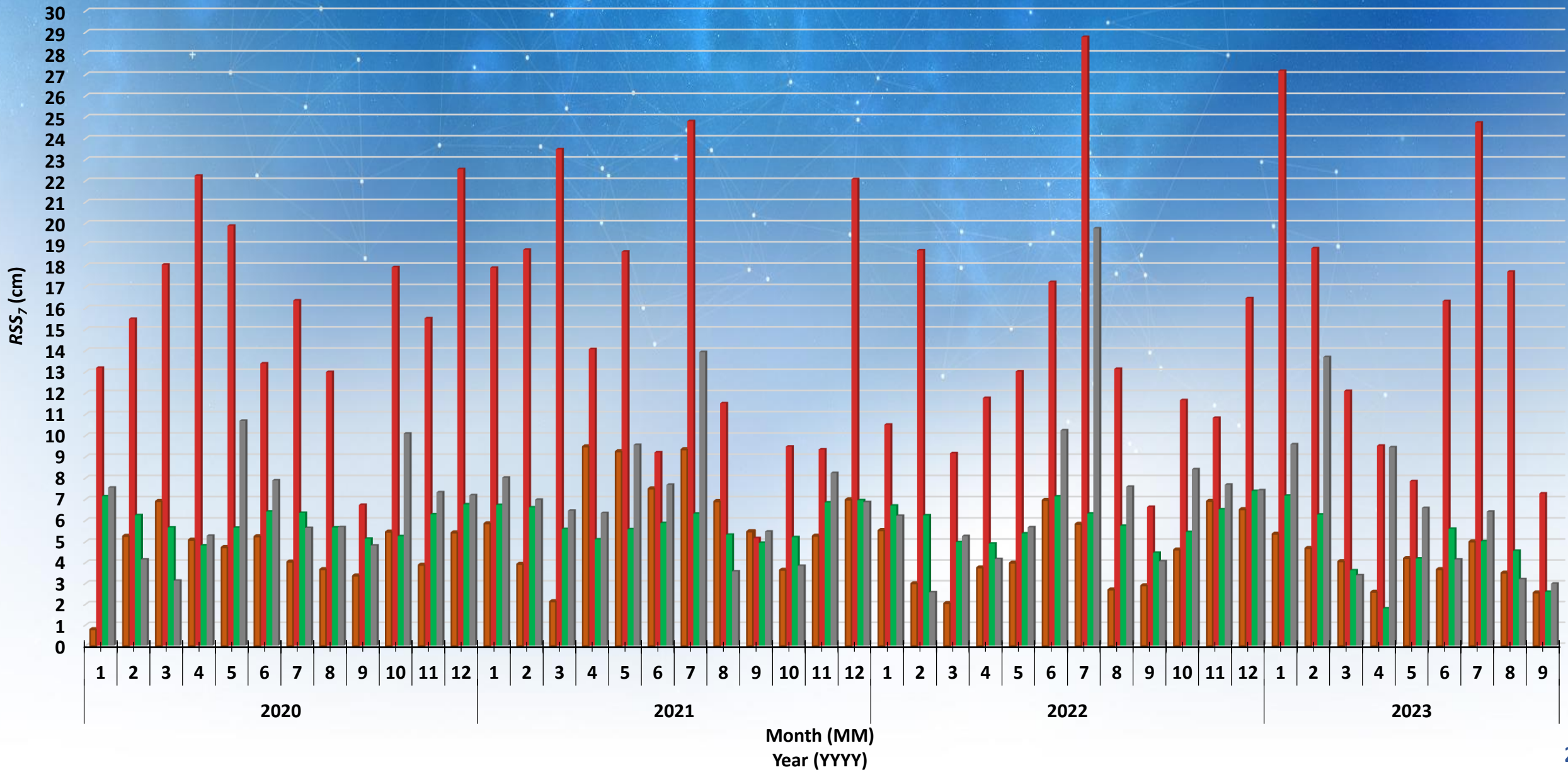
ROSCOSMOS

Data period: 01/01/2020 – 30/09/2023

	GNSS	$\sigma_{\Delta X}$ (cm)	$\sigma_{\Delta Y}$ (cm)	$\sigma_{\Delta Z}$ (cm)	σ_{R_X} (mas)	σ_{R_Y} (mas)	σ_{R_Z} (mas)	σ_m (ppb)	StD_{RSS_7} (cm)	StD_{Λ} (cm)
I	Galileo	0,4	0,5	3,7	0,11	0,12	0,24	2,80	**4,3**	**4,2**
II	GPS	0,3	0,3	3,7	0,30	0,56	0,80	2,90	**5,2**	**4,9**
III	BDS	0,3	0,5	4,2	0,81	1,39	1,27	0,19	**7,6**	**6,7**
IV	GLONASS	0,3	0,4	4,5	0,22	0,32	3,65	0,71	**12,2**	**10,3**

*S. Malys et al. (2021) Compatibility of Terrestrial Reference Frames used in GNSS broadcast messages during an 8 week period of 2019. Advances in Space Research 67 (2021) 834–844

Monthly repeatability in terms of RSS_7 among GNSS TRFs and ITRF2020 during 2020-2023



Final results

	Annual coincidence	Monthly repeatability
I	WGS 84/GPS (1-3 cm)	GTRF/Galileo (4 cm)
II	BDCS/BeiDou (2-4 cm)	WGS 84/GPS (5 cm)
III	GTRF/Galileo (2-5 cm)	BDCS/BeiDou (7-8 cm)
IV	PZ-90/GLONASS (9-15 cm)	PZ-90/GLONASS (10-12 cm)

Conclusions

- Since 2020, IAC PNT has evaluated the coincidence level between terrestrial reference frames used in GNSS broadcast messages. Annual coincidence and monthly repeatability of GNSS TRFs were estimated based on daily solutions for all valid space vehicles, taking into account the measurements from around 1000 globally distributed ground stations.
- The coincidence level among GNSS TRF and ITFR2020 was assessed in terms of RSS_7 and Λ. The results obtained show that RSS_7 is almost equal to Λ.
- The best coincidence with the ITRF2020 is demonstrated by WGS 84 (GPS), the second and third places are held by BDCS (BeiDou) and GTRF (Galileo), PZ-90 (GLONASS) is in the fourth place. The coincidence between the current realization of PZ-90 (PZ-90.11) and the latest version of ITRF (ITRF2020) is at the level of 10 cm, which generally meets the specified tactical and technical requirements for GLONASS. At the same time, the coincidence between WGS 84 (GPS) and ITRF is assessed as 1-3 cm, BDCS (BeiDou) – 2-4 cm, and GTRF (Galileo) – 2-5 cm.
- The assessment of monthly repeatability shows that the GTRF (Galileo) is the most stable in terms of coincidence with ITRF2020, the second and third places are taken by WGS 84 (GPS) and BDCS (BeiDou) respectively, PZ-90 (GLONASS) is in the fourth place.