How does GNSS positioning depend on interoperability?

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Interoperability problems

- What is interoperability?
 - PNT services of dual or multiple GNSS systems can be used together to provide better capabilities at the user level than that would be achieved by relying sorely on a single system or service

Interoperability factors

- Signal-in-space:
 - All generate codes and carrier phase measurements, carrying data messages
 - Each system transmits signals at different frequencies
 - Different frequencies introduce different inter-frequency biases
- Satellite clocks of different systems
 - Are maintained by different groups of ground clocks
- Reference Systems
 - With same or different definitions and parameters
- Reference Frames
 - Different realisation of the same or different reference systems

What "better capabilities"?

- Better accuracy
 - Position (absolute or relative)
 - Clock (absolute or relative)
- Higher availability
- Improved integrity (multiple failures etc)
- Better AR reliability (high success rates)
- Sharper timeliness
 - Shorter time to first fix
 - Shorter time to convergence
- Longer baseline/wider coverage
- More ionospheric/tropospheric profiles?
- etc

Observational Models

GPS L1, L2, L5	Galileo, L1,E6, E5A, E5B
$P_{L1,i}^{j} = \widetilde{\rho} + I_{L1} + T_{g}^{j} + \varepsilon_{PL1}$	$P_{L1,i}^{j} = \widetilde{\rho} + I_{L1} + T_{g}^{j} + \varepsilon_{PL1}$
$P_{L2,i}^{j} = \tilde{\rho} + \alpha I_{L1} + \alpha T_{g}^{j} + R_{2i} + \varepsilon_{PL2}$	$P_{E6,i}^{j} = \widetilde{\rho} + \alpha I_{L1} + \alpha T_{g}^{j} + R_{E6i} + \varepsilon_{PE6}$
$P_{L5,i}^{j} = \tilde{\rho} + \beta I_{L1} + \beta T_{g}^{j} + R_{5i} + \varepsilon_{PL5}$	$P_{E5a,i}^{j} = \widetilde{\rho} + \beta I_{L1} + \beta T_{g}^{j} + R_{5ai} + \varepsilon_{PE5a}$
	$P_{E5b,i}^{j} = \widetilde{\rho} + \gamma I_{L1} + \gamma \Gamma_{g}^{j} + R_{5bi} + \varepsilon_{PE5b}$
$\phi_{L1,i}^{j} = \widetilde{\rho} - I_{L1} + T_g^{j} + \lambda_1 N_1 + \varepsilon_{L1}$	$\phi_{L1,i}^{j} = \widetilde{\rho} - I_{L1} + T_g^{j} + \lambda_1 N_1 + \varepsilon_{L1}$
$\phi_{L2,i}^{j} = \widetilde{\rho} - \alpha I_{L1} + \alpha T_{g}^{j} + \lambda_{2} N_{2} + R_{2i} + \varepsilon_{L2}$	$\phi_{E6,i}^{j} = \tilde{\rho} - \alpha I_{L1} + \alpha T_{g}^{j} + \lambda_{E6} N_{E6} + R_{E6i} + \varepsilon_{e6}$
$\phi_{L5,i}^{j} = \tilde{\rho} - \beta I_{L1} + \beta T_{g}^{j} + \lambda_{5} N_{5} + R_{5i} + \varepsilon_{L5}$	$\phi_{E5b,i}^{j} = \tilde{\rho} - \beta I_{L1} + \beta T_{g}^{j} + \lambda_{E5a} N_{e5a} + R_{e5ai} + \varepsilon_{e5a}$
	$\phi_{\text{E5b,i}}^{j} = \widetilde{\rho} - \gamma I_{\text{L1}} + \gamma T_{\text{g}}^{j} + \lambda_{\text{E5b}} N_{\text{E5b}} + R_{\text{e5bi}} + \varepsilon_{\text{e5b}}$
$\tilde{\rho} = \rho + c(dt - dT) + d_{trop} + d_{orb}$	$\tilde{\rho} = \rho + c(dt - dT) + d_{trop} + d_{orb}$
$\alpha = (f_1/f_2)^2 = (77/60)^2;$	$\alpha = (f_1/f_{E6})^2 = (154/125)^2$
$\beta = (f_1/f_5)^2 = (154/115)^2$	$\beta = (f_1/f_{e5a})^2 = (154/118)^2$
	$\gamma = (f_1/f_{e5b})^2 = (154/115)^2$

Single Point Positioning Model

$$\begin{bmatrix} P_{a} \\ P_{b} \end{bmatrix} = \begin{bmatrix} A_{a} \\ A_{b}R_{ba} \end{bmatrix} \delta X + \begin{bmatrix} cJ & 0 \\ 0 & cJ \end{bmatrix} \begin{bmatrix} \delta T_{a} \\ \delta T_{b} \end{bmatrix} + \begin{bmatrix} \epsilon_{a} \\ \epsilon_{b} \end{bmatrix}$$

$$\delta X \equiv \delta X_a$$
 $\delta Xb = R_{ba}\delta X_a$

$$E(\begin{bmatrix} \boldsymbol{\varepsilon}_{a} \\ \boldsymbol{\varepsilon}_{a} \end{bmatrix}) = 0; \qquad Cov(\begin{bmatrix} \boldsymbol{\varepsilon}_{a} \\ \boldsymbol{\varepsilon}_{a} \end{bmatrix}) = \begin{bmatrix} \boldsymbol{\sigma}_{a}^{2} \boldsymbol{Q}_{a} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_{b}^{2} \boldsymbol{Q}_{b} \end{bmatrix} = \boldsymbol{\sigma}_{a}^{2} \boldsymbol{Q}$$

- (1) Determine the same position vector, but different clock biases
- (2) Important to balance between two code variance components for better positioning accuracy
- (3) Q_a and Q_b ideally include the effects of SV orbital variance matrix

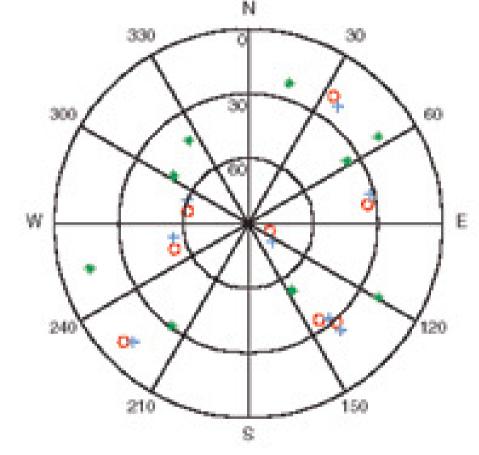
Double Differenced Models

$$\begin{bmatrix} \Delta P_a \\ \Delta \varphi_a \\ \Delta P_b \\ \Delta \varphi_b \end{bmatrix} = \begin{bmatrix} A_a & 0 & 0 \\ A_a & -\lambda_a I_a & 0 \\ A_b R_{ba} & 0 & 0 \\ A_b R_{ba} & 0 & -\lambda_b I_b \end{bmatrix} \begin{bmatrix} \delta X \\ \Delta N_a \\ \Delta N_b \end{bmatrix} + \begin{bmatrix} \epsilon_{\Delta P a} \\ \epsilon_{\Delta \varphi a} \\ \epsilon_{\Delta P b} \\ \epsilon_{\Delta \varphi b} \end{bmatrix}$$

$$E\begin{pmatrix} \boldsymbol{\epsilon}_{\Delta Pa} \\ \boldsymbol{\epsilon}_{\Delta \varphi a} \\ \boldsymbol{\epsilon}_{\Delta Pb} \\ \boldsymbol{\epsilon}_{\Delta \varphi a} \end{pmatrix}) = 0; \qquad Cov \begin{pmatrix} \boldsymbol{\epsilon}_{\Delta Pa} \\ \boldsymbol{\epsilon}_{\Delta \varphi a} \\ \boldsymbol{\epsilon}_{\Delta Pb} \\ \boldsymbol{\epsilon}_{\Delta \varphi b} \end{pmatrix} = \begin{bmatrix} \boldsymbol{\sigma}_{a}^{2} \boldsymbol{Q}_{a} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_{\varphi a}^{2} \boldsymbol{Q}_{\varphi a} \end{bmatrix} \otimes \boldsymbol{D}_{a} \boldsymbol{D}_{a}^{T} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_{\varphi a}^{2} \boldsymbol{Q}_{\varphi a} \end{bmatrix} \otimes \boldsymbol{D}_{b} \boldsymbol{D}_{b}^{T} \\ \boldsymbol{0} & \boldsymbol{0} & \begin{bmatrix} \boldsymbol{\sigma}_{b}^{2} \boldsymbol{Q}_{b} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_{\varphi b}^{2} \boldsymbol{Q}_{\varphi b} \end{bmatrix} \otimes \boldsymbol{D}_{b} \boldsymbol{D}_{b}^{T} \end{bmatrix}$$

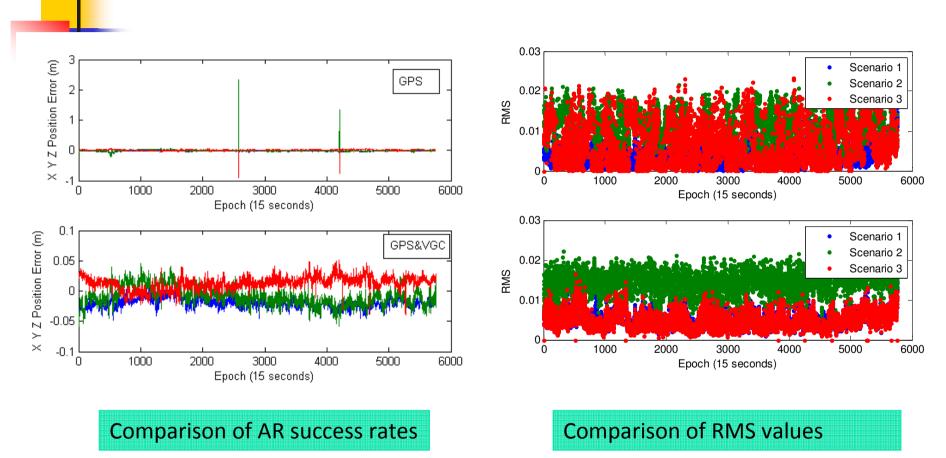
- 1. No cross-constellation differencing avoids requirements for a same frequency
- 2. Balancing between the variance components of different systems is important for better RTK performance

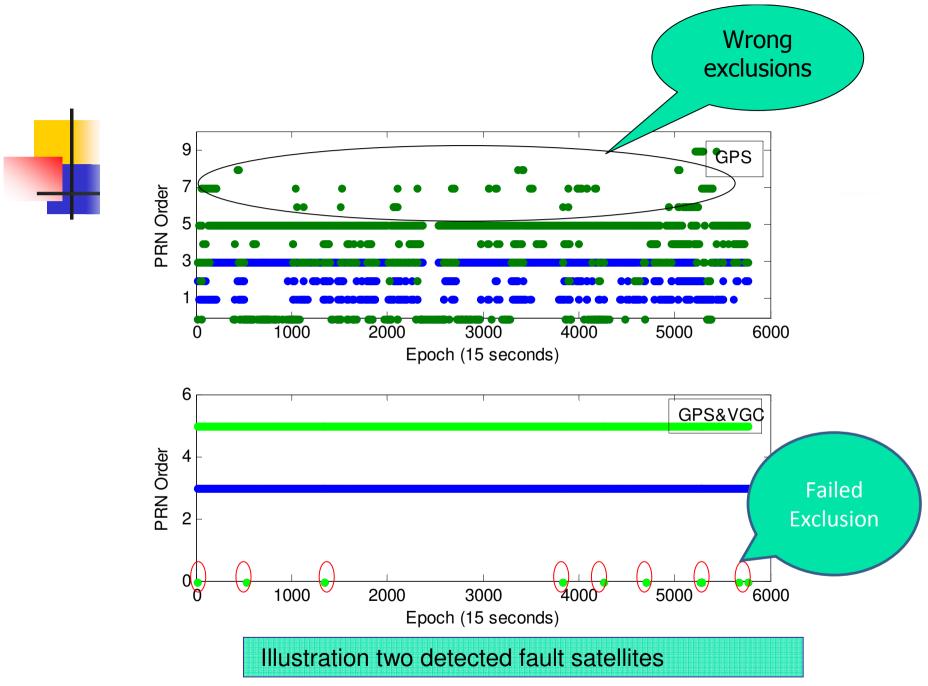
Virtual Galileo Constellation



$$V(\varepsilon_{Pa}) = AS_x A_a^T + \sigma_{Trop}^2 + \sigma_{iono}^2 + \sigma_{multipath}^2 + \sigma_{ea}^2$$

AR success rate and RMS values





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	Yes/No	Confirmed
Better position accuracy?	Yes	Yes
Better clock accuracy?	Not sure	Not Yet
Higher availability?	Yes	Yes
Improved integrity	Yes	Yes
Better AR reliability?	Yes	Yes
Sharper timeliness	Yes	Not Yet
Longer baseline/wider coverage	Yes	Not Yet
More atmospheric soundings	Yes	Yes
More ionospheric samples	Yes	Yes

Concluding remarks

- Observational models and estimation algorithms can be flexibly designed to achieve most of better capabilities without additional requirements for interoperability
- It would be beneficial to real time users if each system broadcasts SV variance matrices